

Mathematical Modeling of Transport of Pollutants in Unsaturated Porous Media with Radioactive Decay and Comparison with Soil Column Experiment

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Abstract--- Most of the investigators use the coordinate transformation ($z - ut$) in order to solve the equation for dispersion of a moving fluid in porous media. Further, the boundary conditions $C = 0$ at $z = 0$ and $C = C_0$ at $z = -L$ for $t \geq 0$ are used, which results in a symmetrical concentration distribution. In this paper, the effect of radioactive tracer has been analyzed for one-dimensional transport of pollutants through the unsaturated porous media and compared with experimental data. In this study, the advection-dispersion equation is solved analytically to evaluate the transport of pollutants which takes into account the decay of radioactive contaminants by considering input concentrations of pollutants that vary with time and depth. The solution is obtained with Laplace transform and moving coordinates to reduce linear partial differential equation to ordinary differential equation and Duhamel's theorem is used to get the solution in terms of complementary error function and verified with experimental data.

Keywords--- Advection, Dispersion, Isotopes, Integral Transforms, Fick's Law, Moving Coordinates, Duhamel's Theorem

I. INTRODUCTION

In recent years considerable interest and attention have been directed to dispersion phenomena in flow through porous media. Scheidegger (1954), deJong (1958), and Day (1956) have presented statistical means to establish the concentration distribution and the dispersion coefficient. Advection-diffusion equation describes the solute transport due to combined effect of diffusion and convection in a medium. It is a partial differential equation of parabolic type, derived on the principle of conservation of mass using Fick's law. Due to the growing surface and subsurface hydro environment degradation and the air pollution, the advection-diffusion equation has drawn significant attention of hydrologists, civil engineers and mathematical modelers. Its analytical/numerical solutions along with an initial condition and two boundary conditions help to understand the contaminant or pollutant concentration distribution behavior through an open medium like air, rivers, lakes and porous medium like aquifer, on the basis of which remedial processes to reduce or eliminate the damages may be enforced. It has wide applications in other disciplines too, like soil physics, petroleum engineering, chemical engineering and biosciences.

In the initial works while obtaining the analytical solutions of dispersion problems in ideal conditions, the basic approach was to reduce the advection-diffusion equation into a diffusion equation by eliminating the convective term(s). It was done either by introducing moving co-ordinates (Ogata and Banks 1961; Harleman and

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Rumer 1963; Bear 1972; Guvanasen and Volker 1983; Aral and Liao 1996; Marshal *et al* 1996) or by introducing another dependent variable (Banks and Ali 1964 Ogata 1970; Lai and Jurinak 1971; Marino 1974 and Al-Niami and Rushton 1977). Then Laplace transformation technique has been used to get desired solutions.

Some of the one-dimensional solutions have been given (Tracy 1995) by transforming the non-linear advection–diffusion equation into a linear one for specific forms of the moisture content *vs.* pressure head and relative hydraulic conductivity *vs.* pressure head curves which allow both two-dimensional and three-dimensional solutions to be derived. A method has been given to solve the transport equations for a kinetically adsorbing solute in a porous medium with spatially varying velocity field and dispersion coefficients (Van Kooten 1996).

Later it has been shown that some large subsurface formations exhibit variable dispersivity properties, either as a function of time or as a function of distance (Matheron and deMarsily 1980; Sposito *et al* 1986; Gelhar *et al* 1992). Analytical solutions were developed for describing the transport of dissolved substances in heterogeneous semi infinite porous media with a distance dependent dispersion of exponential nature along the uniform flow (Yates 1990, 1992). The temporal moment solution for one dimensional advective-dispersive solute transport with linear equilibrium sorption and first order degradation for time pulse sources has been applied to analyze soil column experimental data (Pang *et al* 2003). An analytical approach was developed for non-equilibrium transport of reactive solutes in the unsaturated zone during an infiltration–redistribution cycle (Severino and Indelman 2004).

The solute is transported by advection and obeys linear kinetics. Analytical solutions were presented for solute transport in rivers including the effects of transient storage and first order decay (Smedt 2006). Pore flow velocity was assumed to be a non-divergence, free, unsteady and non-stationary random function of space and time for ground

water contaminant transport in a heterogeneous media (Sirin 2006). A two-dimensional semi-analytical solution was presented to analyze stream–aquifer interactions in a coastal aquifer where groundwater level responds to tidal effects (Kim *et al* 2007).

A more direct method is presented here for solving the differential equation governing the process of dispersion. It is assumed that the porous medium is homogeneous and isotropic and that no mass transfer occurs between the solid and liquid phases. It is assumed also that the solute transport, across any fixed plane, due to microscopic velocity variations in the flow tubes, may be quantitatively expressed as the product of a dispersion coefficient and the concentration gradient. The flow in the medium is assumed to be unidirectional and the average velocity is taken to be constant throughout the length of the flow field. In this paper, the solutions are obtained for two solute dispersion problems in a longitudinal finite length, respectively. In the first problem time dependent solute dispersion of increasing or decreasing nature along a uniform flow through a homogeneous domain is studied. The input condition is of uniform and varying nature, respectively.

II. MATHEMATICAL FORMULATION AND MODEL

We consider one-dimensional unsteady flow through the semi-infinite unsaturated porous media in the x-z plane in the presence of a toxic material. The uniform flow is in the z-direction. The medium is assumed to be isotropic and homogeneous so that all physical quantities are assumed to be constant. Initially the concentration of the contaminant in the media is assumed to be zero and a constant source of concentration of strength C_0 exists at the surface. The velocity of the groundwater is assumed to be constant. With these assumptions the basic equation governing the flow is

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial z^2} - u \frac{\partial C}{\partial z} - \lambda C \quad (1)$$

Where C is the constituent concentration in the soil solution, t is the time in minutes, D is the hydrodynamic dispersion coefficient, z is the depth, u is the average pore-water velocity and λ is the radioactive decay (Chemical reaction term).

Initially saturated flow of fluid of concentration, $C = 0$, takes place in the medium. At $t = 0$, the concentration of the plane source is instantaneously changed to $C = C_0$. Then the initial and boundary conditions (Fig. 1) for a semi-infinite column and for a step input are

$$\left. \begin{aligned} C(z, 0) &= 0; & z \geq 0 \\ C(0, t) &= C_0; & t \geq 0 \\ C(\infty, t) &= 0; & t \geq 0 \end{aligned} \right\} \quad (2)$$

The problem then is to characterize the concentration as a function of x and t .

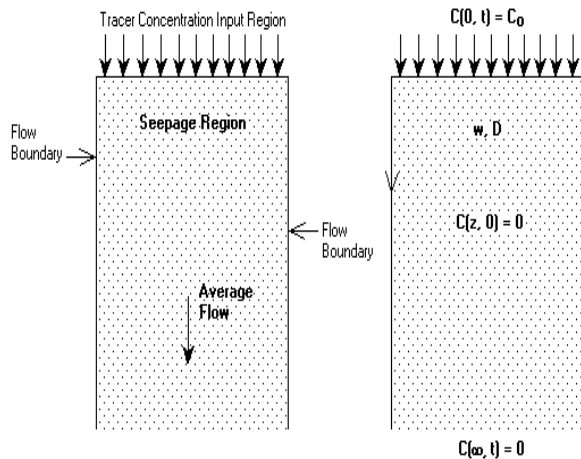


Figure 1: Physical Layout of the Model

To reduce equation (1) to a more familiar form, let

$$C(z, t) = \Gamma(z, t) \exp \left[\frac{uz}{2D} - \frac{u^2 t}{4D} - \lambda t \right] \quad (3)$$

Substitution of equation (3) reduces equation (1) to Fick's law of diffusion equation

$$\frac{\partial \Gamma}{\partial t} = D \frac{\partial^2 \Gamma}{\partial z^2} \quad (4)$$

The above initial and boundary conditions (2) transform to

$$\left. \begin{aligned} \Gamma(0, t) &= C_0 \exp \left(\frac{u^2 t}{4D} + \lambda \right); & t \geq 0 \\ C(z, 0) &= 0; & z \geq 0 \\ C(\infty, t) &= 0; & t \geq 0 \end{aligned} \right\} \quad (5)$$

It is thus required that equation (4) be solved for a time dependent influx of fluid at $z = 0$. The solution of equation (4) can be obtained by using Duhamel's theorem [Carslaw and Jeager, 1947].

If $C = F(x, y, z, t)$ is the solution of the diffusion equation for semi-infinite media in which the initial concentration is zero and its surface is maintained at concentration unity, then the solution of the problem in which the surface is maintained at temperature $\phi(t)$ is

$$C = \int \phi(\tau) \frac{\partial}{\partial t} F(x, y, z, t - \tau) d\lambda$$

This theorem is used principally for heat conduction problems, but above has been specialized to fit this specific case of interest.

Consider now the problem in which initial concentration is zero and the boundary is maintained at concentration unity. The boundary conditions are

$$\left. \begin{aligned} \Gamma(0, t) &= 0; & t \geq 0 \\ \Gamma(x, 0) &= 0; & x \geq 0 \\ \Gamma(\infty, t) &= 0; & t \geq 0 \end{aligned} \right\}$$

This problem can be solved by the application of the Laplace transform. The concentration Γ which is function of t and whatever space coordinates, say z , t , occur in the problem. We write

$$\bar{\Gamma}(z, p) = \int_0^{\infty} e^{-pt} \Gamma(z, t) dt$$

Hence, if equation (4) is multiplied by e^{-pt} and integrated term by term it is reduced to an ordinary differential equation

$$\frac{d^2\bar{\Gamma}}{dz^2} = \frac{p}{D} \bar{\Gamma} \quad (6)$$

The solution of the equation (6) can be written as

$$\bar{\Gamma} = A e^{-qz} + B e^{qz}$$

Where $q = \sqrt{\frac{p}{D}}$.

The boundary condition as $z \rightarrow \infty$ requires that $B = 0$ and boundary condition at $z = 0$ requires that $A = 1/p$, thus the particular solution of the Laplace transform equation is

$$\bar{\Gamma} = \frac{1}{p} e^{-qz}$$

The inversion of the above function is given in a table of Laplace transforms (Carslaw and Jaeger, 1947). The result is

$$\Gamma = 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right) = \frac{2}{\sqrt{\pi}} \int_{\frac{z}{2\sqrt{Dt}}}^{\infty} e^{-\eta^2} d\eta. \quad (7)$$

Utilizing Duhamel's theorem, the solution of the problem with initial concentration zero and the time dependent surface condition at $z = 0$ is

$$\Gamma = \int_0^t \phi(\tau) \frac{\partial}{\partial t} \left[\frac{2}{\sqrt{\pi}} \int_{\frac{z}{2\sqrt{Dt}}}^{\infty} e^{-\eta^2} d\eta \right] d\tau$$

Since $e^{-\eta^2}$ is a continuous function, it is possible differentiate under the integral, which gives

$$\frac{2}{\sqrt{\pi}} \frac{\partial}{\partial t} \int_{\frac{z}{2\sqrt{D(t-\tau)}}}^{\infty} e^{-\eta^2} d\eta = \frac{z}{2\sqrt{\pi D}(t-\tau)^{3/2}} e^{-z^2/4D(t-\tau)}.$$

The solution of the problems is

$$I = \int_0^{\alpha} \exp\left(-\mu^2 - \frac{\varepsilon^2}{\mu^2}\right) d\mu = \frac{1}{2} \left\{ e^{2\varepsilon} \int_0^{\alpha} \exp\left[-\left(\mu + \frac{\varepsilon}{\mu}\right)^2\right] d\mu + e^{-2\varepsilon} \int_0^{\alpha} \exp\left[-\left(\mu - \frac{\varepsilon}{\mu}\right)^2\right] d\mu \right\} \quad (10)$$

$$\Gamma = \frac{z}{\pi\sqrt{D}} \int_0^t \phi(\tau) e^{-z^2/4D(t-\tau)} \frac{d\tau}{(t-\tau)^{3/2}}$$

Letting

$$\mu = \frac{z}{2\sqrt{D(t-\tau)}}$$

The solution can be written as

$$\Gamma = \frac{2}{\sqrt{\pi}} \int_{\frac{z}{2\sqrt{Dt}}}^{\infty} \phi\left(t - \frac{z^2}{4D\mu^2}\right) e^{-\mu^2} d\lambda. \quad (8)$$

Since $\phi(t) = C_0 \exp\left(\frac{u^2 t}{4D} + \lambda t\right)$ the particular solution of the problem can be written as

$$\Gamma(z, t) = \frac{2C_0}{\sqrt{\pi}} e^{\left(\frac{u^2 + \lambda}{4D}\right)t} \left\{ \int_0^{\alpha} \exp\left(-\mu^2 - \frac{\varepsilon^2}{\mu^2}\right) d\mu - \int_0^{\alpha} \exp\left(-\mu^2 - \frac{\varepsilon^2}{\mu^2}\right) d\mu \right\} \quad (9)$$

Where $\varepsilon = \sqrt{\left(\frac{u^2}{4D} + \lambda\right)} \frac{z}{2\sqrt{D}}$ and $\alpha = \frac{z}{2\sqrt{Dt}}$.

A. Evaluation of the Integral Solution

The integration of the first term of equation (9) gives (Pierce, 1956)

$$\int_0^{\infty} e^{-\mu^2 - \frac{\varepsilon^2}{\mu^2}} d\lambda = \frac{\sqrt{\pi}}{2} e^{-2\varepsilon}$$

For convenience the second integral can be expressed in terms of error function (Horenstein, 1945), because this function is well tabulated. Noting that

$$\begin{aligned} -\mu^2 - \frac{\varepsilon^2}{\mu^2} &= -\left(\mu + \frac{\varepsilon}{\mu}\right)^2 + 2\varepsilon \\ &= -\left(\mu - \frac{\varepsilon}{\mu}\right)^2 - 2\varepsilon \end{aligned}$$

The second integral of equation (9) can be written as

Since the method of reducing integral to a tabulated function is the same for both integrals in the right side of equation (10), only the first term is considered. Let $a = \frac{\varepsilon}{\mu}$ and adding and subtracting, we get

$$e^{2\varepsilon} \int_{\varepsilon/\alpha}^{\infty} \exp \left[- \left(a + \frac{\varepsilon}{a} \right)^2 \right] da.$$

The integral can be expressed as

$$I = e^{2\varepsilon} \int_0^{\alpha} \exp \left[- \left(\mu + \frac{\varepsilon}{\mu} \right)^2 \right] d\mu = -e^{2\varepsilon} \int_{\varepsilon/\alpha}^{\infty} \left(1 - \frac{\varepsilon}{a^2} \right) \cdot \exp \left[- \left(\frac{\varepsilon}{a} + a \right)^2 \right] da + e^{2\varepsilon} \int_{\varepsilon/\alpha}^{\infty} \exp \left[- \left(\frac{\varepsilon}{a} + a \right)^2 \right] da.$$

Further, let

$$\beta = \left(\frac{\varepsilon}{a} + a \right)$$

In the first term of the above equation, then

$$I_1 = -e^{2\varepsilon} \int_{\alpha + \frac{\varepsilon}{\alpha}}^{\infty} e^{-\beta^2} d\beta + e^{2\varepsilon} \int_{\varepsilon/\alpha}^{\infty} \exp \left[- \left(\frac{\varepsilon}{a} + a \right)^2 \right] da.$$

$$\Gamma(z, t) = \frac{2C_0}{\sqrt{\pi}} e^{\left(\frac{u^2}{4D} + \lambda \right) t} \left\{ \frac{\sqrt{\pi}}{2} e^{-2\varepsilon} - \frac{1}{2} \left[e^{-2\varepsilon} \int_{\frac{\varepsilon}{\alpha} - \alpha}^{\infty} e^{-\beta^2} d\beta - e^{2\varepsilon} \int_{\frac{\varepsilon}{\alpha} + \alpha}^{\infty} e^{-\beta^2} d\beta \right] \right\} \quad (11)$$

However, by definition

$$e^{2\varepsilon} \int_{\alpha + \frac{\varepsilon}{\alpha}}^{\infty} e^{-\beta^2} d\beta = \frac{\sqrt{\pi}}{2} e^{2\varepsilon} \operatorname{erfc} \left(\alpha + \frac{\varepsilon}{\alpha} \right)$$

Also,

$$e^{-2\varepsilon} \int_{\frac{\varepsilon}{\alpha} - \alpha}^{\infty} e^{-\beta^2} d\beta = \frac{\sqrt{\pi}}{2} e^{-2\varepsilon} \left[1 + \operatorname{erf} \left(\alpha - \frac{\varepsilon}{\alpha} \right) \right]$$

Writing equation (11) in terms of the error functions, we get

$$\frac{C}{C_0} = \frac{1}{2} \exp \left[\frac{uz}{2D} \right] \left[e^{2\varepsilon} \operatorname{erfc} \left(\alpha + \frac{\varepsilon}{\alpha} \right) + e^{-2\varepsilon} \operatorname{erfc} \left(\alpha - \frac{\varepsilon}{\alpha} \right) \right] \quad (12)$$

Resubstituting the value of ε and α gives

Similar evaluation of the second integral of equation (10) gives

$$I_2 = e^{-2\varepsilon} \int_{\varepsilon/\alpha}^{\infty} \exp \left[- \left(\frac{\varepsilon}{a} - a \right)^2 \right] da - e^{-2\varepsilon} \int_{\varepsilon/\alpha}^{\infty} \exp \left[- \left(\frac{\varepsilon}{a} - a \right)^2 \right] da.$$

Again substituting $-\beta = \frac{\varepsilon}{a} - a$ into the first term, the result is

$$I_2 = e^{-2\varepsilon} \int_{\frac{\varepsilon}{\alpha} - \alpha}^{\infty} e^{-\beta^2} d\beta - e^{-2\varepsilon} \int_{\varepsilon/\alpha}^{\infty} \exp \left[- \left(\frac{\varepsilon}{a} - a \right)^2 \right] da.$$

Noting that

$$\int_{\varepsilon/\alpha}^{\infty} \exp \left[- \left(a + \frac{\varepsilon}{a} \right)^2 + 2\varepsilon \right] da = \int_{\varepsilon/\alpha}^{\infty} \exp \left[- \left(\frac{\varepsilon}{a} - a \right)^2 - 2\varepsilon \right] da$$

Substitute this into equation (10) gives

$$I = e^{-2\varepsilon} \int_{\frac{\varepsilon}{\alpha} - \alpha}^{\infty} e^{-\beta^2} d\beta - e^{2\varepsilon} \int_{\frac{\varepsilon}{\alpha} + \alpha}^{\infty} e^{-\beta^2} d\beta.$$

Thus, equation (9) can be expressed as

$$\Gamma(z, t) = \frac{C_0}{2} e^{\left(\frac{u^2}{4D} + \lambda \right) t} \left[e^{2\varepsilon} \operatorname{erfc} \left(\alpha + \frac{\varepsilon}{\alpha} \right) + e^{-2\varepsilon} \operatorname{erfc} \left(\alpha - \frac{\varepsilon}{\alpha} \right) \right]$$

Substitute the value of $\Gamma(z, t)$ in equation (3) the solution reduces to

$$\frac{C}{C_0} = \frac{1}{2} \exp\left[\frac{uz}{2D}\right] \left[\exp\left[\frac{\sqrt{u^2 + 4D\lambda}}{2D} z\right] \cdot \operatorname{erfc}\left(\frac{z + \sqrt{u^2 + 4D\lambda}}{2\sqrt{Dt}} t\right) + \exp\left[-\frac{\sqrt{u^2 + 4D\lambda}}{2D} z\right] \cdot \operatorname{erfc}\left(\frac{z - \sqrt{u^2 + 4D\lambda}}{2\sqrt{Dt}} t\right) \right] \quad (13)$$

Where boundaries are symmetrical the solution of the problem is given by the first term of equation (13). The second term in equation (13) is thus due to the asymmetric boundary imposed in a general problem. However, it should be noted that if a point a great distance away from the source is considered, then it is possible to appropriate the boundary conditions by $C(-\infty, t) = C_0$, which leads to a symmetrical solution.

III. SOIL COLUMN STUDIES IN THE LABORATORY

In the field of agriculture, the retention of nitrogen in surface soils is of vital concern in the application of plant fertilizers. The most notable cause of nitrogen contamination of groundwater is the release of wastewater at or near the surface of the earth. Preul and Schroepfer (1968) have investigated the effects resulting from intimate contact between nitrogen-bearing solutions and selected soils. Column tests were devised in the laboratory keeping the following variables under control: soils, concentration and forms of influent nitrogen, rate of flow application, temperature, and soil-solution contact time. The specific purpose of the column tests were:

The clay soil, which possesses largest surface area per volume of soil, showed the highest adsorptive capacity whereas the sand, having lower surface area per volume of soil, showed the lowest capacity. The adsorption process is largely a surface-action phenomenon.

As series of tests was performed to determine the effect of having various concentrations of the potassium along with ammonium. Potassium is shown to be quite similar to ammonium in its ion-exchange characteristics. The amount

of adsorption decreased with increased concentrations of K, as per the isotherms got. The potassium cat ions replaced the ammonium ions as K was increased.

The movement of nitrate ion through soil is of utmost importance since, out of all the nutrient ions, it is the most susceptible to loss by leaching and because of its potential impact on water quality. In agriculture, when fertilizers are applied, nitrogen after satisfying the crop requirement accumulates in the soil medium and changes the natural soil equilibrium. This results in the accumulation of nitrogen (i.e., NH_4^+ or NO_3^-) and behaves as a pollutant (Hajek, 1969).

Many investigators have carried out experiments on disturbed soil columns with chloride as tracer (McMahon and Thomas, 1974). Chloride has been shown to be a good tracer of nitrate. Therefore in leaching studies chloride can be used without biological interference; it can be assumed that nitrate behaves similarly under well-aerated conditions. According to experimental work by Thomas and Swoboda (1970) and Smith (1972) on disturbed soil columns, the chloride moved deeper into the soil. The disturbed columns do not adequately describe leaching as experienced under field conditions. Natural drainage in the undisturbed column would tend to setup vertical channels and pathways while the disturbed columns would be more homogeneous and the chloride might move through more nearly as a front.

Sadashivaiah et al., (2000) have assessed the values of hydrodynamic dispersion D, based on experiments carried out on soil columns of three study sites using artificial nitrate solution and tritium as tracer.

A. Laboratory Tests

The experimental investigation was carried out on two different undisturbed soil columns extracted from two different study sites (Thirnahalli and Bhairanayakanahalli, Chikkaballapur) by the help of a specially devised core attached with a mild steel shoe (plate 2).

The soil columns were of 30cm height and 10cm diameter, and extracted from the unsaturated zones, by piercing a rigid PVC pipe of 10cm internal diameter, with the shoe. Two such columns were extracted from each of the two study sites.

The objectives of the experiments are:

- to predict the hydrodynamic dispersion coefficient, and
- to study the behavioral characteristics of the pollutants during their travel through soils.

The soil columns were mounted on to a rigid plywood stand fabricated for the purpose (Fig. 2).

The tracer solutions employed were-deionized water, aqueous potassium nitrate and tritium (tritiated water) and used separately for three different columns extracted from each site. The potassium nitrate influent solution was prepared by dissolving dry KNO_3 chemical in deionized water to obtain a concentration of 25mg/l as NO_3^- . Initially the tritium counts were 3290 per minute in the tritiated water (tracer) prepared for experimentation on soil column from Bhairanayakanahalli, Chikkaballapur site.

The soils from the study sites were sampled and subjected to essential physical and chemical tests using standard test procedures (Jackson, 1958). The results of analysis are presented in Tables (1) – (11).

The water samples from the nearby wells were sampled periodically and monitored for important chemical parameters. The results of analyses are tabulated in Tables (1) – (11).

B. Methodology for Experimentation

The deionized water was allowed to flow through the soil column to study the leaching level of NO_3^- and timely variations in the soil water interactions appearing as lechates. Different cations and anions, pH, EC, metals etc., were analysed adhering to the standard procedures (Eaton, 1995). The variations of important parameters, viz., nitrate, chloride and pH are presented in Figure 2 for the given study sites.

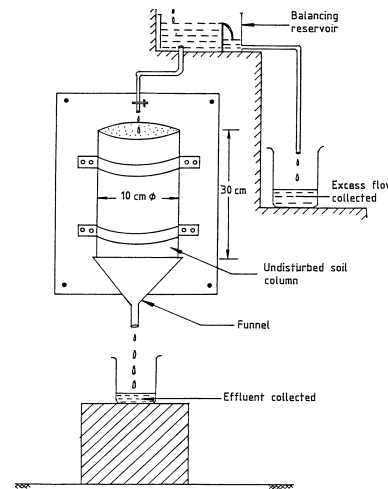


Figure 2: Schematic View of Vertical Soil Column Experiment Setup

Results of chemical analysis of soil samples carried out at M/s Zuari Agro-chemicals Laboratory, Bangalore, before and after passing nitrate aqueous solution through 30cm length and 10cm diameter soil column extracted from Bhairanayakanahalli site are in the following tables.

Table 1: Physical Properties of Soil (November 1997)

Sl. No.	Physical Properties of Soil	Thirnahalli	Bhairanayakanahalli
1.	Specific gravity	2.5	2.55
2.	Moisture content, %	15	18
3.	Bulk density, gm/cc	1.9	2.05
4.	Dry density, gm/cc	1.65	1.73
5.	Void ratio, e	0.51	0.47
6.	Degree of saturation, S %	73	97
7.	Porosity, n %	33.7	32
8.	Permeability, cm/day	1.44	1.41

Table 2: Physical Properties of Soil (January 1998)

Sl. No.	Physical Properties of Soil	Thirnahalli	Bhairanayakanahalli
1.	Specific gravity	2.615	2.564
2.	Moisture content, %	14.26	14.84
3.	Bulk density, gm/cc	1.829	2.033
4.	Dry density, gm/cc	1.601	1.77
5.	Void ratio, <i>e</i>	0.6332	0.4485
6.	Degree of saturation, <i>S</i> %	58.89	84.83
7.	Porosity, <i>n</i> %	38.77	30.96
8.	Permeability, cm/day	1.44	0.427

Table 3: Physical Properties of Soil (July 1998)

Sl. No.	Physical Properties of Soil	Thirnahalli	Bhairanayakanahalli
1.	Specific gravity	2.60	2.39
2.	Moisture content, %	7.85	4.85
3.	Bulk density, gm/cc	1.96	2.0
4.	Dry density, gm/cc	1.810	1.90
5.	Void ratio, <i>e</i>	0.43	0.25
6.	Degree of saturation, <i>S</i> %	46.76	46.36
7.	Porosity, <i>n</i> %	30	0.20
8.	Permeability, cm/day	2.15	1.90

Table 4: Chemical Properties of Soil (November 1997)

Sites	pH	EC mS/cm	OC %	P ₂ O ₅ Kg/acre	K ₂ O Kg/acre	Zn ppm	Fe ppm	Cu ppm	Mn ppm	Ca %
Thirnahalli	7.2	0.07	0.25	4.4	79.2	2.0	3.1	0.5	5.1	0.05
Bhairanayakanahalli	7.1	0.08	0.33	3.3	61.6	2.3	2.0	0.8	5.2	0.03

Table 5: Chemical Properties of Soil of Thirnahalli (March 1998)

Sl. No.	Depth wise (cm)	pH	EC mS/cm	OC %	P ₂ O ₅ Kg/acre	K ₂ O Kg/acre	Zn ppm	Fe ppm
1.	0	5.9	0.02	0.35	9	92	1.6	30.9
2.	30	6.0	0.02	0.11	2	58	0.4	17.1
3.	60	6.5	0.02	0.11	2	56	0.33	10.1
4.	90	6.9	0.02	0.07	2	60	0.15	9.0

Table 6: Chemical Properties of Soil of Thirnahalli (July 1998)

Sl. No.	Depth wise (cm)	pH	EC mS/cm 25°	OC %	Available Phosphorous Kg/acre	Available Potash Kg/acre	Available Zn ppm
1.	0	5.9	0.03	0.4L	4L	146H	1.6S
2.	45	5.3	0.05	0.24L	2L	98M	0.41D

Table 7: Chemical Properties of Soil of Bhairanayakanahalli (March 1998)

Sl. No.	Depth wise (cm)	pH	EC mS/cm	OC %	P ₂ O ₅ Kg/acre	K ₂ O Kg/acre	Zn ppm	Fe ppm
1.	0	6.0	0.07	0.23	2	54	1.10	57.2
2.	30	6.3	0.05	0.11	8	60	0.53	37.9
3.	60	5.9	0.06	0.11	16	120	0.44	37.0
4.	90	6.8	0.05	0.23	4	180	0.31	12.1

Table 8: Analysis of Water Samples (January 1998)

StudySite	pH	EC	CO ₃ ⁻	HCO ₃ ⁻	Cl	SO ₄ ⁻	Ca ⁺	Ca ⁺⁺	Mg ⁺⁺	Na ⁺	SAR
Thirnahalli	8.0	1.1	0.3	3.4	6.9	--	6.9	4.8	2.1	4.2	2.2
Bhairanayakanahalli	8.4	0.8	--	4.6	3.2	--	4.6	3.0	1.6	3.6	2.3

Table 9: Analysis of Water Samples (March 1998)

Study Site	pH	EC μmho/cm	Total Hardness Mg/l as CaCO ₃ ⁻	Cl ⁻ mg/l as Cl ⁻	SO ₄ ²⁻ mg/l	K mg/l	PO ₄ ²⁻ mg/l	Na ⁺ mg/l	NO ₃ mg/l
Thirnahalli	7.85	690	240	90	16	18.9	1.7	50.5	23.5
Bhairanayakanahalli	7.64	675	320	70	17	4.4	0.86	32.0	28.6

Table 10: Analysis of Water Samples (August 1998)

Study Site	pH	EC μmho/cm	Total Hardness Mg/l as CaCO ₃ ⁻	Cl ⁻ mg/l as Cl ⁻	SO ₄ ²⁻ mg/l	K mg/l	PO ₄ ²⁻ mg/l	Na ⁺ mg/l	NO ₃ mg/l
Thirnahalli	7.96	780	190	77	19	21.87	1.915	58.05	13.23
Bhairanayakanahalli	7.34	720	300	60	18	1.08	0.810	28.06	18.01

Table 11: Analysis of Water Samples (January 1998)

Sl. No	Study Site	Na mg/l	K mg/l	Ammonical Nitrogen mg/l	Nitrates NO ₃ mg/l	Phosphates mg/l
1.	Thirnahalli	78	27	Not detectable	177	0.04
2.	Bhairanayakanahalli	58	0.2	Not detectable	177	0.0044

The input to the soil columns is drawn from the constant level tank containing the tracer and allowed over the column drop by drop to avoid ponding of the tracer on the column as detailed in the Table 12 below.

Table 12: Analysis of Water Samples (January 1998)

Study Sites	Types of Soil	Input rate (ml/hr)	
		NO ₃ tracer	Deionized water
Thirnahalli	Well graded sandy soil	106	77
Bhairanayakanahalli	Uniformly graded sandy soil	105	76

The tracer occupies the pores present in the soil column and so to say, the soil becomes saturated. Then the tracer comes out as leachate from the bottom end of the soil column. The time elapsed between the first drop of leachate coming out of the bottom of the soil column and the time at the input to the column top is recorded in all the three soil columns used for experimental runs.

Table 13: Analysis of Water Samples (January 1998)

Sl. No.	Study Sites	Time elapsed	
		NO ₃ tracer	Deionized water
1.	Thirnahalli	30 min	50 min
2.	Bhairanayakanahalli	30 min	50 min

For the third soil column tritiated water was used as the tracer. Among several unstable radioactive isotopes, tritium is considered to be an ideal tracer in many hydrological investigations.

Tritiated water (HTO) is an important tracer which belongs to the group of conservative tracers under artificial isotropic water tracers, having a half life of 12.43 years and emits low energy β -radiation ($E_{\max}=17.6$ kev) (Vasu and Hameed, 1998).

The tritium tracer was allowed to pass through the third soil column at a uniform velocity. The activity in the leachate samples were assessed by injecting a volume of 1 ml of sample diluted to 3 ml of distilled water and 6 ml of diaxane cocktail. The mixture in 20 ml vials were churned vigorously. The vails were then arranged in liquid scintillation machine (plate 3) and the activity counts were obtained. Tritium being a nonadsorbing ideal tracer, is widely adopted in predicting the migration mechanics of the pollutants in natural soil.

The Break-Through-Curves (BTC) was constructed for artificial nitrate tracer (Fig. 3&4) and for tritium tracer (Fig. 5&6).

Making use of the experimental data related to soil columns with NO₃ and tritium tracers, the dispersion coefficient values were computed using Fried and Cumbarous (1971) equation, viz.,

$$D = \frac{1}{8} \left[\frac{z - vt_{0.16}}{\sqrt{t_{0.16}}} - \frac{z - vt_{0.84}}{\sqrt{t_{0.84}}} \right]^2$$

Where $t_{0.16}$ and $t_{0.84}$ are the times required (in minutes) for the concentration ratios of $C/C_0=0.16$ and $C/C_0=0.84$ respectively to reach a particular distance z . The results are given in Table 14.

Table 14: Dispersion Coefficient Values

Study sites	Nitrate tracer			Tritium tracer		
	$t_{0.16}$ (min)	$t_{0.84}$ (min)	D m ² /yr	$t_{0.16}$ (min)	$t_{0.84}$ (min)	D m ² /yr
Thirna halli	600	3360	11.24	150	1240	85.51
Bhaira nayaka nahalli	600	3360	11.24	150	1240	85.51

IV. RESULTS AND DISCUSSION

In water resource development, one of the major problems encountered is that of water quality. The sources of groundwater pollution can be divided into four major groups such as, environmental, domestic, industrial and agricultural. Groundwater contains salts carried in solution, which is added due to rainwater, irrigation, artificial recharge, soluble rock minerals, fertilizers, etc. Accidental breaking of the sewers and leakages from septic tank may also increase the level of pollution. Contaminants are often leaked from chemical and petrochemical plants and also from waste deposits. When radioactive wastes are buried at great depths, the only fluid media that can possibly interact with it is the groundwater.

In modern agricultural activity large quantities of water soluble fertilizers are frequently applied to the soil surface. A portion of them remains in the root zone, and the rest is carried underground by the moving water. To estimate the magnitude of the hazard posed by some of these chemicals, it is important to investigate the processes that control their movement from the soil surface through the root zone down to the groundwater table. At present, major thrust on the transport of contaminant and research is directed towards the definition and quantification of the process governing the behaviour of pollutants in sub surface environment, coupled with the development of mathematical models that integrate process descriptions with the pollutant properties and site characteristics. Furthermore, a contaminant after coming in contact with the soil, can undergo several bio-geo-chemical processes viz., sorption-desorption, transformation/ degradation and leaching.

The present investigation describes theoretical considerations and presents tools for analyzing solute conditions during infiltration from a source. The partial differential equations describing solute transport are solved analytically and numerically. It is generally assumed that macroscopic transport by convection must take into account the average flow velocity as well as the mechanical or hydrodynamic dispersion. Taking into account the scope of the work, the investigation carried out in the field as well as in the laboratory, the observations made and the results obtained are presented under different sections as mentioned below:

Water containing pollutants such as sewage and radioactive wastes, infiltrate through the soil matrix from streams, ditches and lakes as well as direct flow from overland areas as result of runoff. This water eventually enters the groundwater storage basin (aquifer) – a source for potable water. During the passage of water through the soil, the pollutants are mixed, dispersed and diffused through the flowing flux and led to an intense effort to develop more accurate and economical models for predicting solute transport and fate, often from solute sources that exist in the unsaturated soil zone.

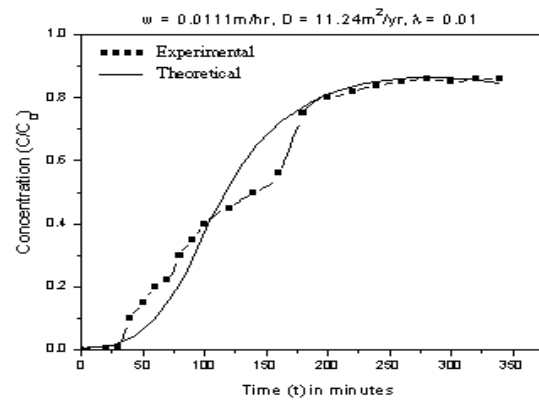
The mixing takes place in the soil medium by two processes, viz., molecular diffusion and dispersion. Molecular diffusion is a physical process, which depends upon the kinetic properties of the fluid particles and cause mixing at the contact front between the two fluids. Dispersion, however, is defined as mechanical mixing process caused by the tortuous path followed by the fluid flowing in the geometrically complex interconnections of the flow channels and by the variations in equations describing solute transport are solved analytically and numerically. Analytical solutions for one– dimensional model is obtained using Laplace transformation techniques.

The main limitations of the analytical method are, that the applicability is for relatively simple problems. The geometry of the problem should be regular. The properties

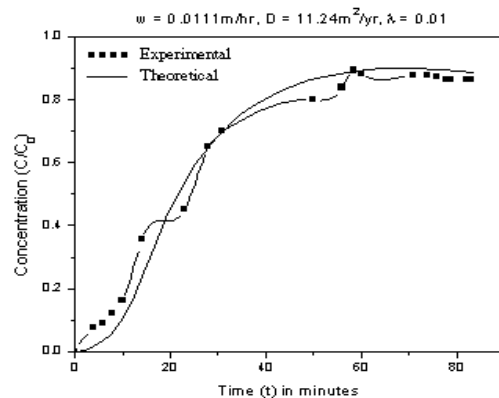
of the soil in the region considered must be homogeneous or at least homogeneous in the sub region. The analytical method is somewhat more flexible than the standard form of other methods for one – dimensional transport model.

From the equation (13) C/C_0 was numerically computed using 'Mathematica' and the results are presented graphically in figures. Figures represent the Break-Through-Curves for C/C_0 vs time for different depth z . It is seen that the concentration field increases in the beginning and reaches a steady state value for a fixed z but decreases with an increase in the radioactive decay coefficient λ . An increase in λ will make the solute concentration decreasing as evident from the physical grounds.

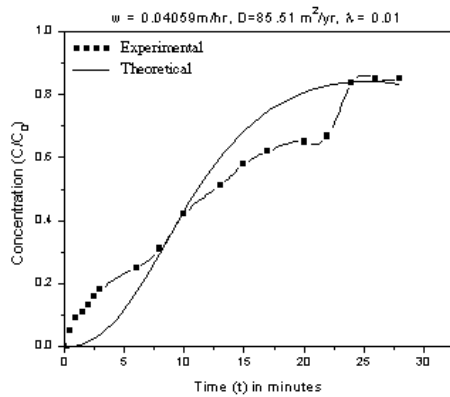
Figures show the comparison of analytic solution with those of the experimental ones and are found to be in good agreement.



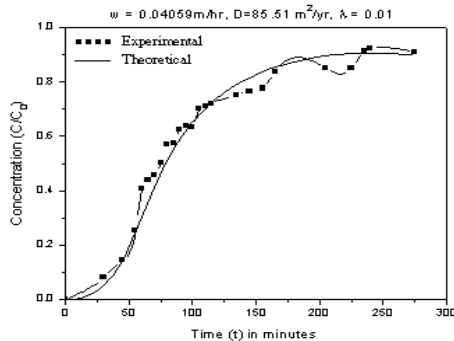
KNO_3 as a Pollutant, Thirnahalli



KNO_3 as a Pollutant, Bhairanayakanahalli



Tritium Tracer as a Pollutant, Thirnahalli



Tritium Tracer as a Pollutant, Bhairanayakanahalli

Collection and analyses of soil samples and undisturbed soil columns from the experimental sites, estimation of the parameters required for developing the models and the laboratory investigations and the parameters computed there from are presented. The following observations are also made:

- When tritiated water was passed through separate sets of soil columns in different dispersion characteristics were observed. The sample from Bhairanayakanahalli site has a dispersion coefficient as high as $85.51 \text{ m}^2/\text{year}$; Thirnahalli site: $11.24 \text{ m}^2/\text{year}$, indicating a good dispersion of applied chemical constituents and thereby earlier contamination of groundwater due to applied chemical species. However, the soil column of Nelamangala site had a lower dispersion coefficient of $17.87 \text{ m}^2/\text{year}$ with tritium tracer, indicating relatively slower migration of contaminants.

- The soils of Bhairanayakanahalli site and Thirnahalli site (Kolar District) responded for a lesser moisture holding capacity between 2.71 to 7.86% and 6.36 to 15.17%, when the suction in pressure was 15 bars and 0.3 bars, respectively, (the moisture between the field capacity 0.3bars and wilting point 15 bars is available for crop). Hence, the crops in this region would require frequent irrigation to supplement and maintain the required moisture by the crops. The irrigation requirement of these soils is relatively less for the crops. These results would help the farmers, planners and decision makers in effective management of irrigation water. This will also help in the possibilities of applied chemicals leaching through over irrigation and thereby resulting in groundwater contamination by fertilizers.

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