# Aerodynamic Characteristics of a Single Slotted Raked Wingtip at Different Reynolds Number

Sangram K Samal and Dilip A Shah

Abstract--- Effects of Reynolds number on tip vortex characteristics of two different wing geometries at three Reynolds numbers  $1.82 \times 10^5$ ,  $3.12 \times 10^5$  and  $4.16 \times 10^5$ have been studied in this investigation for various angles of attack. A sweptback tapered wing and a slotted raked wingtip were the two configurations considered for this study. The parameter which changed the most was the free stream velocity to obtain different Reynolds numbers keeping other parameters constant. The numerical results obtained for tangential velocity, lift coefficient and drag coefficient of sweptback tapered NACA 0015 wing were compared with the experimental results to validate the CFD code. The numerical results that were obtained by using CFD code show a good agreement with the experimental results. The effect of Reynolds number on non-dimensional aerodynamic parameters was noticed at higher angles of attack. The effect was visible only at angles of attack greater than 8° for lift coefficient and for drag coefficient after 6° angle of attack. Maximum C<sub>L</sub> and stalling angle were increased with Reynolds number for both the configurations but the drag coefficient was found to be decreased with increase in Reynolds number. But the dimensional aerodynamic parameters were found to increase with the Reynolds number. It was also observed that the aerodynamic efficiency of slotted raked wingtip was higher compared to the sweptback tapered wing configuration for all the three Reynolds numbers.

**Keywords**--- Tip-vortex, Raked, Slotted, Circulation, Wingtip, Reynolds Number

## NOMENCLATURE

u <sub>c</sub>	Core axial velocity
$\mathbf{u}_{\infty}$	Free stream velocity
$v_{\theta}$	Tangential velocity
$v_{\theta peak}$	Peak tangential velocity
х	Streamwise direction
у	Normal to wing direction
Z	Spanwise direction
Γ	Circulation or vortex strength
$\Gamma_{\rm c}$	Core Circulation
$\Gamma_{\rm o}$	Total Circulation
Re	Reynolds number
Re λ	Reynolds number Taper ratio (ct/cr)
	•
λ	Taper ratio (ct/cr)
$\lambda$ c <sub>r</sub>	Taper ratio (ct/cr) Root chord
$\lambda$ $c_r$ $c_t$	Taper ratio (ct/cr) Root chord Tip chord
$\lambda$ $c_r$ $c_t$ AR	Taper ratio (ct/cr) Root chord Tip chord Aspect Ratio
λ $c_r$ $c_t$ AR α	Taper ratio (ct/cr) Root chord Tip chord Aspect Ratio Angle of Attack
λ $c_r$ $c_t$ AR α $α_{Stall}$	Taper ratio (ct/cr) Root chord Tip chord Aspect Ratio Angle of Attack Stalling Angle
λ $c_r$ $c_t$ AR α $α_{Stall}$ $C_L$	Taper ratio (ct/cr) Root chord Tip chord Aspect Ratio Angle of Attack Stalling Angle Lift Coefficient

## I. INTRODUCTION

Viscous effects, as governed by the Reynolds number, have a major influence on the airfoil lift, drag, and other aerodynamic characteristics. Hence it is important to analyse the effect of Reynolds number on wing tip vortex as it may lead to reduction of induced drag. The Reynolds number is a unit less number which allows a full scale aerodynamic structure to be compared with a test model, giving insight as to the forces as well as the aerodynamic characteristics expected from the full scale object.

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Simulations were made for the three different Reynolds numbers  $(1.82 \times 10^5, 3.12 \times 10^5 \text{ and } 4.16 \times 10^5)$ . The main objective of the present work was to see the effects of Reynolds number on wing tip vortex and reduction of induced drag using Slotted Raked Wingtip. The investigation of wing tip vortex is an important task in aerodynamics, because their influence is directly related to induce drag.

In a finite wing, there is an opportunity for the pressures acting on the upper and lower surfaces to interact near the wing tip [1]. The shorter the distance between the wing tip, the larger the downwash velocity and the induced drag [2]. The trailing vortex system also generates an upwash in the regions beyond the wing span and a downwash inside the wing span. This downwash produced by the trailing vortex system adds to the downwash produced by the bound vortex system [3].

Aerodynamic efficiency can be improved by increasing the maximum lift-to-drag ratio at the cruise flight condition. Because induced drag is typically 30 percent or more of the total drag on a subsonic transport in cruising flight [4]. A 10% drag reduction on a large military transport aircraft is estimated to save up to 13 million gallons of fuel over its lifetime [5]. The world's total jet fleet is estimated to be around 17 thousand aircrafts [6]. Such reduction in drag could result in enormous savings in the fuel cost. This drag is even more significant at low speed, during takeoff conditions, where it can account for 80-90% of the aircraft drag [7].

Besides the advantage of lowering operating costs, reducing wingtip shed vorticity, and therefore induced drag, may also reduce global warming because of the lower fuel consumption. The world's commercial jet aircraft generate more than 600 million tons of carbon dioxide per year [8].

The techniques for its tip vortices reduction include winglets, wingtip sails, Raked wing tips and Ogee tips. Much of the development work for the winglet was initiated by Whitcomb at NASA [9,10]. Adding winglets to a wing can reduce and diffuse the vortex structure which originates at the tips [5, 12, 13]. Also wing tip vortex can be reduced by using active means (suction at wing tip) rather than wing tip shaping which are a passive means [14, 15].

The rapid evolution of CFD has been driven by the need for faster and more accurate methods for the calculations of the flow fields around the configurations of technical interest. In the past decade, CFD was the method of choice in the design of many aerospace, automotive and industrial components and processes in which fluid or gas flows play a major role. In the fluid dynamics, there are many commercial CFD packages available for modelling flow in or around objects. Bacha and Ghaly [16] presented a transition model that combined existing methods for predicting the onset and extent of transition, which were compatible with the Spalart-Allmaras turbulence model, where the flow was simulated using Fluent.

This paper presents the validation of CFD code against McGill University experimental data and then to investigate the 3-D flow structure of two different configuration (clean wing and wing with slotted raked wingtip) at Re= $1.81 \times 10^5$ . Special attention was given to the effects of Reynolds number on aerodynamic characteristics of sweptback, tapered and single slotted raked wingtip.

#### **II. PROCEDURE**

# A. Overview

This paper presents a comparison between wingtip vortex flow field carried out by CFD simulations and experimental measurements done at McGill University by Gerontakos and Lee [11] as a validation of the present numerical CFD simulations. Also several different parameters were computed for the two different configurations at different Reynolds numbers and presented in graphical format. The Reynolds number is dependent upon the density of the fluid, the average velocity of the airfoil relative to the fluid, the characteristic length of the airfoil, as well as the dynamic viscosity of the fluid. Reynolds number was changed by changing only the free stream velocity and keeping other parameters constant. It is possible now to predict all aspects of tip-vortices using modern CFD code with commendable accuracy for unswept and swept-back wing [17, 18].

The present simulations were run in ANSYS 13.0, which models fluid flow and heat transfer problems in complex geometries. This commercial CFD software solves the general transport equations using the finite volume method. Steady-state, transient, incompressible, compressible, in-viscid, viscid, laminar, and turbulent flows can be solved with Fluent. The simulations were run for three different Reynolds numbers  $(1.82 \times 10^5, 3.12 \times 10^5 \text{ and } 4.16 \times 10^5)$  by changing only the free stream velocity. The free stream velocities were set to 35 m/s, 60 m/s. and 80 m/s to achieve the corresponding Reynolds number.

#### B. Complete Geometry Case

The near-field flow structure of two different configurations(tapered sweptback wing and wing with slotted raked wingtip) investigated were using Computational Fluid Dynamics (CFD). The baseline geometry (tapered sweptback wing configuration) was a half-wing model (Fig. 1) used by Gerontakos and Lee [11] at McGill University in a low speed wind tunnel. The model was an untwisted sweptback, tapered wing with an aspect ratio of 3.654, a taper ratio of 0.375, a semi-span of 51 cm, and a wing area of 713 cm<sup>2</sup>. The root chord was 20.3 cm and the tip chord was 7.6 cm. This test model was used for three different free stream velocityfor 35 m/s, 60 m/s. and 80 m/s. The locations of the downstream vortex-flow measurement planes (denoted by the dashed lines,  $x/c_r=2$ ) are shown in Fig.1. The sweep angle at 0.25-chord location was set at  $24^{\circ}$ . The square tipped wing had a NACA 0015 section throughout at 8° angle of attack. The second configuration (single slotted raked wingtip) was created from base line configuration. The tapered swept back wing (baseline geometry) was modified to raked wingtip (Fig. 2) by changing only the wing tip geometry. The wing tip was modelled similar to raked wingtip configuration of Boeing

787. Then the raked wingtip geometry was modified keeping a single slot like on bird feather only for raked part as shown in Fig. 3 to create the single slotted raked wingtip configuration.

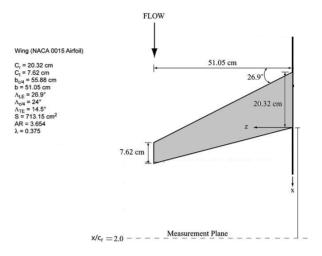


Fig. 1: Baseline Geometry (Tapered Sweptback Wing Configuration)

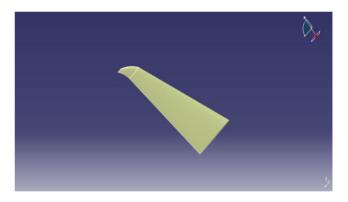


Fig. 2: Raked wingtip (Wing Tip was Modelled Similar to Raked Wingtip Configuration of Boeing 787)

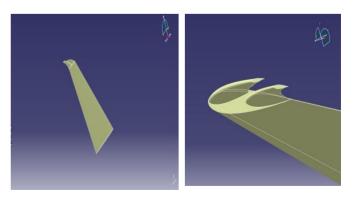


Fig. 3: Single Slotted Raked Wingtip (Second Configuration)

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## **III. RESULTS AND DISCUSSION**

The primary objective of this work was to investigate the effects of Reynolds number on tip vortex characteristics for two different wing geometries (tapered sweptback wing and single slotted raked wingtip configuration) at various angles of attack. The work essentially consists of three sections. The first section validates the CFD code with experimental data. In other two sections the effects of Reynolds number on tip vortex characteristics of sweptback tapered wing and single slotted raked wingtip configuration were studied respectively.

#### A. Validation of the CFD code

Non-dimensional tangential velocity  $(v_{\theta}/u_{\infty})$  distribution about the vortex centreat 8° angle of attack and lift, drag coefficients at different angles of attack were computed at free stream velocity of 35 m/s. These computed values were compared with the experimental measurements done at McGill University by Gerontakos and Lee [11]. Figures 4-6 show a comparison between numerical simulations done in Fluent and experimental result of  $v_{\theta}/u_{\infty}$ ,  $C_L$  and  $C_D$ . The simulations show good agreement with the experimental data.

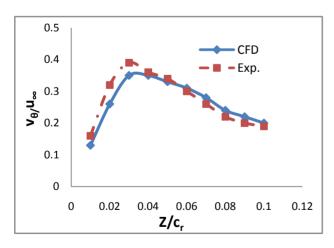


Fig. 4: Non-dimensional Tangential Velocity about the Wingtip Vortex Core

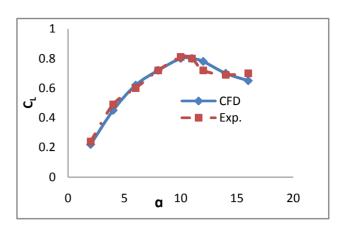


Fig. 5: Lift Coefficients-for Different Angle of Attack

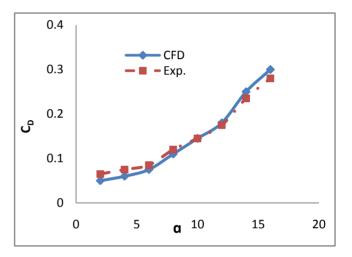


Fig. 6: Drag Coefficients-for Different Angle of Attack

#### B. Investigation of Sweptback Tapered Wing

The circulation, or vortex strength can be calculated either by taking line integral of velocity around a closed curve or by the surface integral over vorticity[19, 20]. The circulation value can be obtained from the surface integral as follows:

$$\Gamma = \oint (udx + vdy) = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} dxdy$$

where equation of vorticity  $(\boldsymbol{\zeta})$  is

$$\frac{\partial \mathbf{v}}{\partial \mathbf{x}} - \frac{\partial \mathbf{u}}{\partial \mathbf{y}} = \boldsymbol{\zeta}$$

The circulation value was verified by using both the line integral and surface integral method. Noticeable difference was not found between these two calculation methods. In this paper line integral method was used to find out the circulation. Several different flow parameters were computed for the three different Reynolds number at different angles of attack and are presented in Figure 7. These indicated that  $v_{\theta peak}$ ,  $\Gamma_c$  and  $\Gamma_o$  were increased with angle of attack whereas  $u_c$  was decreased with angle of attack. The effect of Reynolds number was noticed in the figure 7. The deviation of the flow parameters ( $v_{\theta peak}$ ,  $\Gamma_c$  and

Γ) were more and more as the angle of attack increases for different Reynolds number but in case of u<sub>c</sub> the trend is different. The deviation of u<sub>c</sub> decreases with angle of attack for different Reynolds number. Also the maximum value of v<sub>θpeak</sub>, Γ<sub>c</sub>, Γ<sub>o</sub> and u<sub>c</sub> were increased with Reynolds number.

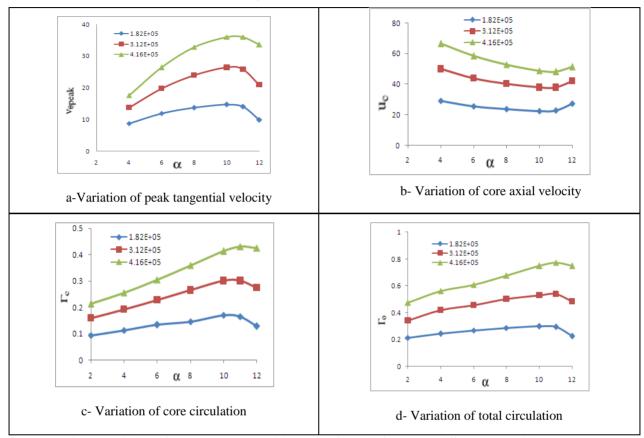
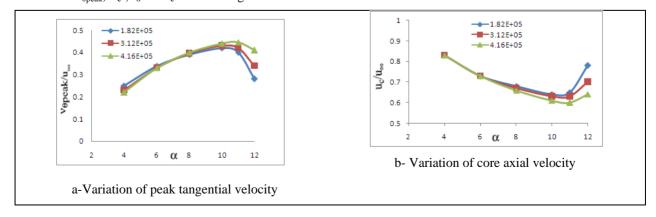


Fig. 7: Variation of Flow Parameters with Angle of Attack for Three Different Reynolds Number

Non-dimensional flow parameters were computed for the three different Reynolds number at different angles of attack and presented in Figure 8. There was no significant difference in  $v_{0peak}$ ,  $\Gamma_c$ ,  $\Gamma_o$  and  $u_c$  at lower angle of attack but the effect of Reynolds number was noticed at higher angle of attack. The flow parameters increase with Reynolds number at higher angle of attack (more than  $8^{\circ}$ ).



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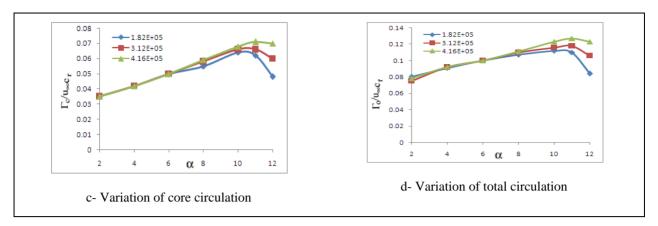


Fig. 8: Non-Dimensional Flow Parameters for the Three Different Reynolds Number at Different Angle of Attack

The values of lift and drag coefficients are calculated between the angles of attack of 2° and 14°. Figures 9 and 10 show the variation of lift coefficient and drag coefficient with angle of attack for the three different Reynolds numbers. There were no significant differences due to the effect of Reynolds number on C<sub>L</sub> till 8° angle of attack. But after 8° angle of attack there were noticeable changes in C<sub>L</sub>- $\alpha$  curve due to different Reynolds number. It was noticed that the maximum C<sub>L</sub> and stalling angle increased with Reynolds number. But in case of drag coefficient the effect of Reynolds number was noticed earlier than lift coefficient (6° angle of attack). Drag coefficient decreased with increase in Reynolds number where as lift coefficient increased with increase in Reynolds number at higher angles of attack. Also aerodynamic efficiency (C<sub>L</sub> / C<sub>D</sub>) was calculated and represented in Figure 11. It gives the variation of aerodynamic efficiency with angle of attack between 2° and 14° for the three different Reynolds number. The effect of Reynolds number on aerodynamic efficiency was not found at lower angle of attack but increased with Reynolds number at higher angles of attack (more than 6 degree).

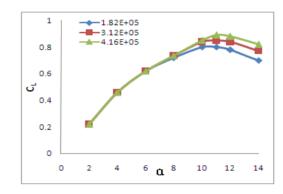


Fig. 9: Variation of Lift Coefficient with Angle of Attack

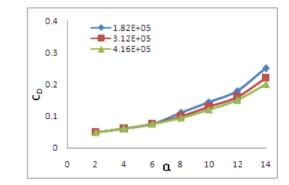


Fig. 10: Variation of Drag Coefficient with Angle of Attack

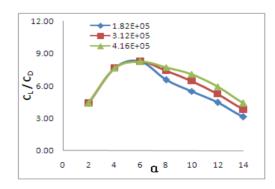
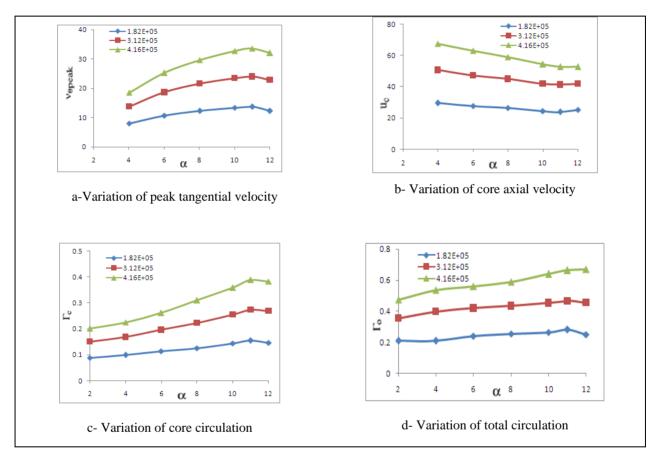
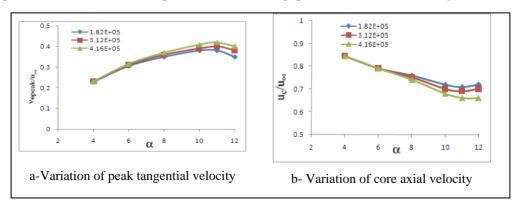


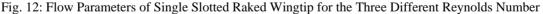
Fig. 11:  $C_L / C_D$  with Angle of Attack

# C. Investigation of Single Slotted Raked Wingtip Configuration

Figure 12 gives the variation of different flow parameters for the three different Reynolds numbers for single slotted raked wingtip configuration. The nondimensional flow parameters were computed for the three different Reynolds number and presented in graphical format in Figure 13. Also lift coefficient, drag coefficient and aerodynamic efficiency were computed for the angles of attack in the range of  $2^{\circ}$  to  $14^{\circ}$  and the variation of these parameters with angle of attack for the three different Reynolds numbers are plotted in figures 14, 15 and 16. All these plots show the similar trend as sweptback tapered wing configuration. The detail comparison between sweptback tapered wing and wing with slotted raked wingtip for three different Reynolds number are given in next section.







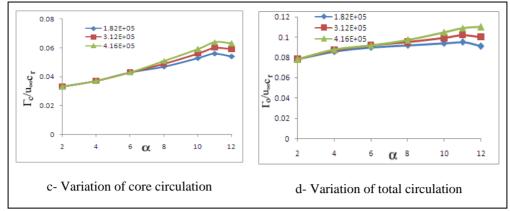


Fig. 13: Non-Dimensional Flow Parameters of Single Slotted Raked Wingtip for the three Different Reynolds Number

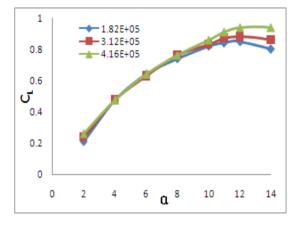


Fig. 14: Variation of Lift Coefficient with Angle of Attack

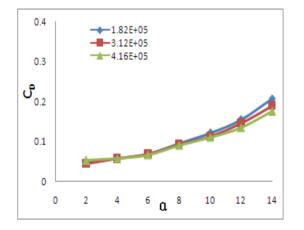


Fig. 15: Variation of Drag Coefficient with Angle of Attack

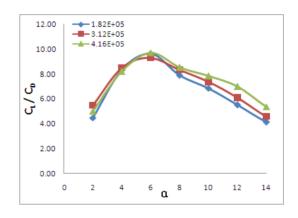
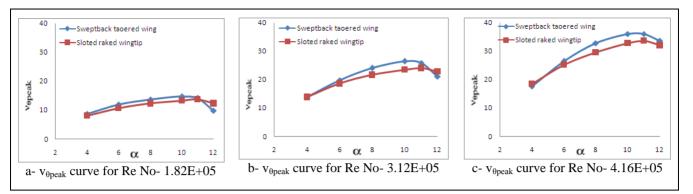


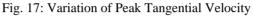
Fig. 16:  $C_L / C_D$  with Angle of Attack

# D. Comparison between Two Configuration (Sweptback Tapered Wing and Wing with Slotted Raked Wingtip)

The comparison of peak tangential velocity, core axial velocity, core circulation and total circulation between sweptback tapered wing and single slotted raked wingtip configuration are presented in figures 17, 18, 19 and 20 for three different Reynolds numbers. For all the three Reynolds number the value of tangential velocity, core circulation and total circulation are higher for sweptback tapered wing as compared to single slotted raked wingtip configuration. But the core axial velocity behaves in opposite way. This value is lower for sweptback tapered wing as compared to single slotted raked wingtip configuration (figure 18).

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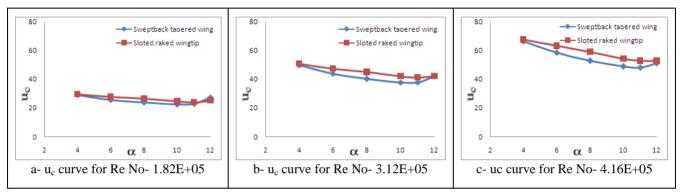
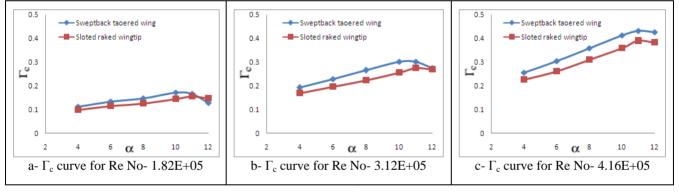
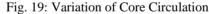


Fig. 18: Variation of Core Axial Velocity





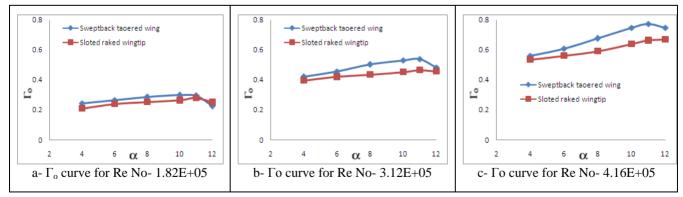
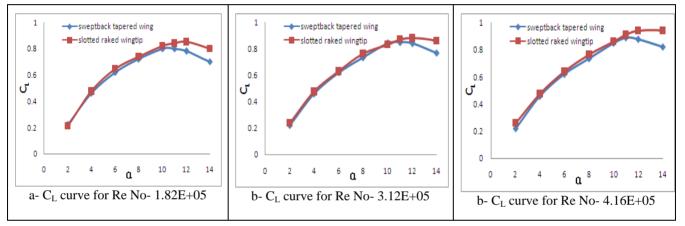


Fig. 20: Variation of total circulation

Figures 21, 22 and 23 give the comparison of lift coefficient, drag coefficient and aerodynamic efficiency between sweptback tapered wing and single slotted raked wingtip configurations for three different Reynolds number. No significant difference was noticed for the  $C_{L}$ -  $\alpha$  curve till  $10^{\circ}$  angle of attack. But after  $10^{\circ}$  angle of attack there were noticeable changes for  $C_{L}$ -  $\alpha$  curve.  $C_{L}$  and  $\alpha_{Stall}$  were increased for single slotted raked wingtip as compared to sweptback tapered wing (figure 21). Figure 22 presents the comparison of drag coefficient at different angles of attack for the three different Reynolds number. There was no significant difference in  $C_{D}$  at lower angle of attack (less than 5°) but the effect of slotted raked wingtip configuration was noticed after 5° angle of attack. Figure 23 gives the comparison of aerodynamic efficiency ( $C_L$  /  $C_D$ ). Aerodynamic efficiency was found to be more for single slotted raked wingtip as compared to sweptback tapered wing configuration. Also difference of aerodynamic efficiency between single slotted raked wingtip and sweptback tapered wing configuration were compared and plotted in figure 24. The percentages of aerodynamic efficiency were found to be increased with the angle of attack for all the three Reynolds number.



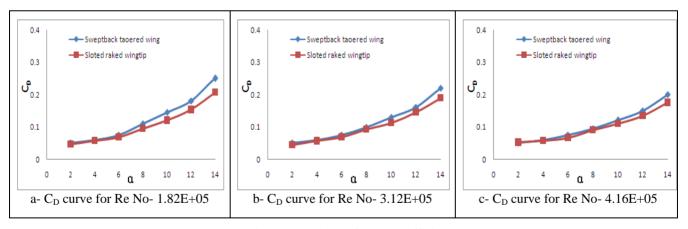


Fig. 21: Variation of Lift Coefficient

Fig. 22: Variation of Drag Coefficient

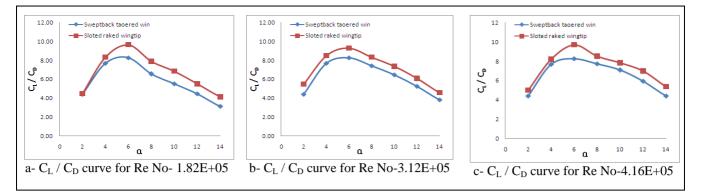


Fig. 23: Variation of Aerodynamic Efficiency  $(C_L / C_D)$ 

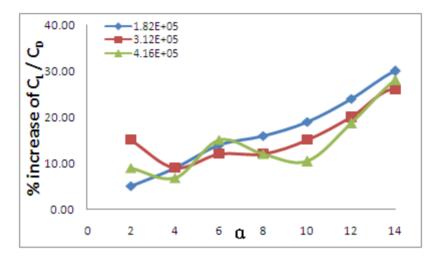


Fig. 24: Increase of Aerodynamic Efficiency for Single Slotted Raked Wingtip with Compare to Sweptback Tapered Wing

# **IV. CONCLUSION**

In the present research work the influence of Reynolds number for two configurations (sweptback tapered wing and the single slotted raked wingtip) were investigated at different angles of attack by using the numerical simulations in FLUENT. The main goal was to study the effect of Reynolds number of the slotted raked wingtip design on the drag reduction.

The flow structure of a tip vortex behind a sweptback and tapered NACA 0015 wing with an AR of 3.654 was investigated for three different Reynolds numbers  $(1.81 \times 10^5, 3.12 \times 10^5 \text{ and } 4.16 \times 10^5)$  at different angles of attack for two different configurations (clean sweptback tapered wing and single slotted raked wingtip). Numerical simulations done in the current study has shown very good agreement with the experimental measurements done by P. Gerontakos and T. Lee at McGill University in predicting the formation of wing tip vortices for a sweptback tapered wing.

The peak tangential velocity, core circulation and total circulation were found to increase initially with angle of attack for all the three Reynolds number up to certain angle of attack (between 10° to 12°) and then tend to decrease for a sweptback tapered wing. In case of single slotted wingtip configuration this angle of attack was higher compared to sweptback tapered wing. But  $u_c$  was found to decrease initially and then increase for all the three Reynolds numbers for both the configurations. There was no significant difference in the non-dimensional parameters such as  $(v_{\theta peak}/u_{\infty})$ ,  $(\Gamma_c/u_{\infty}c_r)$ ,  $(\Gamma_o/u_{\infty}c_r)$  and  $(u_c/u_{\infty})$  at lower angle of attack but the effect of Reynolds number was observed at higher angle of attack (figure 8 and 13) for both the configuration.

The  $C_L$ ,  $C_D$  and  $C_L/C_D$  of both the configurations were computed at the three different Reynolds numbers. The effect of Reynolds number was not observed for  $C_L$  and  $C_D$ up to certain angle of attack. The effect of Reynolds number was noticed after 8° angle of attack for  $C_{L}$ -  $\alpha$  curve and after 6° angle of attack for  $C_D - \alpha$  curve. Maximum  $C_L$  and stalling angle were increased with Reynolds number for both the configurations, but the drag coefficient was found to be decreased with increase in Reynolds number. The effect of Reynolds number on aerodynamic efficiency was observed after 6° angle of attack. Also aerodynamic efficiency was found to be higher for single slotted raked wingtip as compared to sweptback tapered wing configuration for all the three Reynolds number.

It was concluded based on the present study that the effect of Reynolds number for the aerodynamic characteristics found at higher angle of attack. There was no significant effect of Reynolds number at lower angle of attack. Also aerodynamic efficiency was increased using single slotted raked wingtip as compared to clean sweptback tapered wing. For further investigations it would be very helpful to investigate its impact on the very low Reynolds number and compressible flow.

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