Partial Identification Framework for the Identification of Multi-Input Multi-Output System

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Abstract— **A novel concept of partial identification for Multi-Input Multi-Output (MIMO) system under input-output data based modeling is proposed in this work. Issues such as richness in identification data, plant safety, operator's comfort, time involved in identification experiments, identifiability, model accuracy etc. needs to be addressed while conducting an identification experiment. Both open-loop and closed-loop identification methods suffer with one or the other way while considering above issues all together. The proposed partial identification framework helps in taking advantages of both open-loop and closed-loop identification setup on a common platform. The proposed method is applicable to MIMO systems and it uses direct identification method for linear system identification using input-output data. The paper also addresses the potential benefits of a nonlinear regulator/controller and dithering in partial identification framework. The simulation results on MIMO Quadruple Tank system supports the benefits suggested due to partial identification framework.**

I. INTRODUCTION

The purpose of an identification experiment is to determine the dynamics/model of a given process from input-output data. The model thus obtained can be used in various constructive ways for process improvement and control design. Process identification is one of the most important steps of control system design which accounts 80-90% of the cost and time involved. It encompasses a diverse set of task that include plant testing, selection of a model structure, parameter estimation and model validation. Traditionally, identification methods are broadly classified in an open-loop identification and closed-loop identification. Open-loop identification is simple in design. In addition, input-output data are sufficiently rich (informative) for identification of a plant. But in many industrial Multi-Input Multi-Output (MIMO) systems, it is often necessary in practice to perform identification experiments in closed-loop due to safety and/or economic reasons. Beyond this lies the attractive idea of being able to improve the closed-loop performance continually by making use of data being collected from a working loop. In fact, the data collected from the closed-loop operation better reflect the actual situation in which the developed model will be used, and therefore could yield better overall results [1]. Also, in case of an unstable system, it is necessary to perform an identification experiment in closed-loop. Due to above reasons, there has been a rapidly increasing interest on identification of closed-loop systems during last few years [2], [3]. However, on the other hand, a closed-loop condition presents some additional complications for system identification. The fundamental problem is the correlation between the output error/noise and the input through the feedback controller. Because of the correlation, many identification methods that are proven to work with open-loop data can fail. This is true for the prediction error approach as well as the subspace approach and nonparametric approaches like empirical transfer function estimation.

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The fundamental question that arises here is do we need to operate a plant always in completely closed-loop? In a large scale multi-input multi-output scenario, can we benefit from the idea of partially open and partially closed loops for identification exercise? There are potential benefits of conducting partial identification experiments on a large scale MIMO plant. Partial identification has not been dealt extensively in the literature. Although some thoughts have been given in [2], [4]. In this work, we tried to explore the partial identification idea for MIMO systems. The analysis focusses on the bias properties of the plant estimate when applying the direct method of prediction error identification and the possibilities to identify the plant model without the need of simultaneously estimating full-order noise models.

The rest of the paper is organized as follows: Section II focuses on the problem descriptions. Section III presents the feedback-feedforward framework of LQG controller. In section IV simulation results illustrate the robustness and effectiveness of the proposed controller for the SG water level. Finally, Section V concludes the paper.

II. PROBLEM STATEMENT

The identification problem that we seek to address in this paper is shown in Fig. 1. Without loss of generality, consider the case of 3×3 input-output system. As shown in Fig. 1, each input-output combinations are regarded as channel. The problem can also be extended for a large scale multivariable plant where there are many input-output variables present. Let us consider the MIMO plant which consists critical loops, open loops and unstable loops. Critical loop is the loop where input/output variables or both are having bounds and must be within specified limits. Open loop is the loop where input variables are free to manipulate without controller and output variables are not so critical. It is important to note here that the input variables can also be measurable disturbances. Unstable loops comprises of input-output combinations which makes the plant or part of the plant unstable. In this discussion, critical loops, open

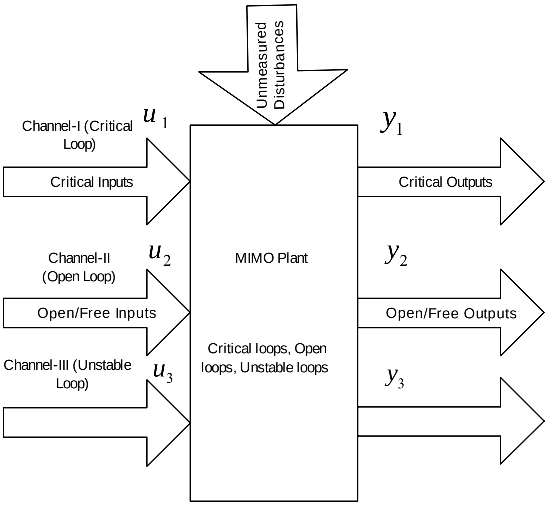

Fig. 1. Schematic of typical MIMO Plant

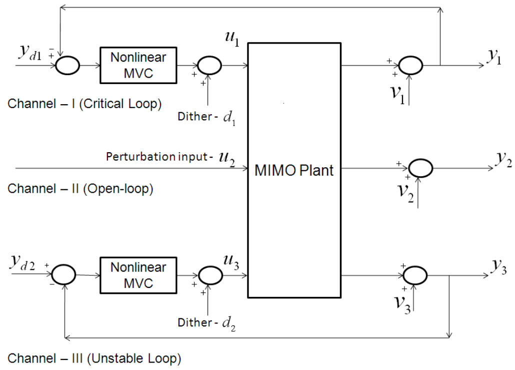

Fig. 2. Block diagram of partial identification setup for MIMO plant

loops and unstable loops are regarded as channel-I, II and III respectively. All indicated signals $u_1, u_2, u_3, y_1, y_2, y_3$ are considered to be multivariate. In addition, there can be unmeasured disturbances which might affect the MIMO plant operation.

In this work, a special case has been considered where the direct method of prediction error identification is chosen for identifying the plant model. As a consequence, a consistent plant model can only be identified if also the full noise model is estimated consistently if a plant is operated in a closed-loop [5]. This is because the feedback induces correlations between plant input and noises. Estimating a full-order plant and noise model for the MIMO plant with large numbers of inputs/outputs, can easily lead to high-dimensional and complex non-convex optimization problems that are hard to solve [2]. A separation of the identification problem can be attractive, where in a first step a plant model is identified and in a second step the noise model is estimated if required, while both models can be validated separately. In an open-loop experiment setup this can be achieved by using independently parameterized plant and noise models. While for closed-loop experiment setup, using the direct identification method for an identification of plant model separately will fail as mentioned above.

Therefore, the problem here is, in multivariable scenario discussed above, how identification can be proposed in a systematic way so as to acquire potential benefits of open-loop and closed-loop identification on common platform.

A. *Details of Partial Identification Framework*

Partial identification is an identification method where part of the MIMO plant loops are open and part of the MIMO plant loops are closed. In this section, a complete framework is given for the partial identification along with some guidelines. Consider the block diagram shown in Fig. 2, which shows the partial identification framework. The details of the framework are presented as follows:

- The framework is proposed based on direct identification method for parameter estimation.
- We assume that unstable modes of the MIMO plant are known apriori. All the unstable loops are required to be operated in closed-loop.
- Critical variables must be chosen carefully as they are required to operate within specified limits. Violating the limits of such variables may directly or indirectly leads to unplanned shutdowns, deterioration of product quality or any significant economic loss. All such loops involving critical variables or affected by critical variables needs to be operated in closed-loop. For example, in a typical drum boiler of a nuclear power plant [6], drum level is a critical output variable because violation of the safety limits of drum level causes unplanned shutdowns.
- The controllers are designed based on apriori knowledge available about the dynamics. The presence of nonlinear controller (that results for example when a model based controller is constrained) helps in minimizing bias errors resulting from input-noise correlation and facilitates the use of the direct method of closed-loop identification.
- In the proposed partial identification problem the lack of informative data is a key problem which arises due to the presence of closed-loop channels. This problem is overcome via the use of a dither signal applied either at the controller output or at the setpoint. The dither signal needs to be carefully designed and implemented to balance the requirements of minimum closed-loop variability as well as richness for identification. If there are more than one interacting channels (closed-loop), the dither in each of them should be designed to be uncorrelated.

B. *Potential Benefits of Partial Identification*

The potential benefits of partial identification framework are briefly summarized as follows:

- It supports unstable plant identification because of the provision of closed-loop configuration for the unstable loops. Open-loop identification is not applicable to unstable plants.

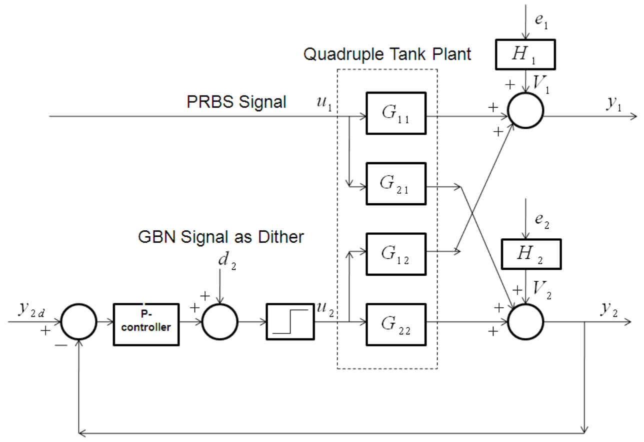

Fig. 3. Partial Identification for Quadruple Tank

- In partial identification, critical variables can not cross their allowable limits because of the presence of controller in critical loops which helps in maintaining same level of statistical accuracy of the plant throughout the identification experiment. The open-loop identification requires attention from the operator continuously during the open-loop identification experiment. In multivariable scenario, manual control can be difficult when many manipulated variables are excited. Operator control will be reduced or even avoided if the process is under feedback control.
- Controller presence in the partially closed loops, will increase the signal-to-noise ratio and thus, reduces the effect of disturbances.
- The closed-loop identification does not allow the manipulated variable perturbations and therefore, the richness in the input-output data is lacking in this case. Instead, partial identification helps in maintaining data richness partially for some of the loops which are open. For other closed-loops, data can be made informative by adding appropriate dither signals.
- The identification experimental time is also shorter than the open-loop identification experiment time.

III. Simulation Results and Discussion

Figure 3 shows partial identification schematic for the quadruple tank process. The details of the schematic are as follows:

Quadruple Tank plant shown here is the discrete time transfer function model for the minimum phase case as described in [9]. The highest open-loop settling time is $t_s = 350$ seconds. The sampling time is chosen as $T_s = \frac{t_s}{100} = 3.5$ seconds. Quadruple Tank considered here is the 2 -Input and 2 -Output process. The levels of the two bottom tanks are the output variables and pump inputs are considered as input variables. It is a four state model where levels of all four tanks are considered as state variables. The absolute range of the inputs are from 0 to 5 $Volts$ and outputs are from 0 to 20 cm.

- The discrete time transfer functions for the quadruple tank plant are as follows:

$$G_{11} = \frac{0.1427q^{-1}}{1-0.9451q^{-1}},\ G_{12} = \frac{0.006013q^{-1}+0.005609q^{-2}}{1-1.804q^{-1}+0.8117q^{-2}},$$
$$G_{21} = \frac{0.003016q^{-1}+0.002864q^{-2}}{1-1.852q^{-1}+0.8559q^{-2}},\ G_{22} = \frac{0.1068q^{-1}}{1-0.9619q^{-1}}$$

- The noise transfer functions are monic, stable and inversely stable as shown below:

$$H_{11} = H_1 = \frac{1}{1+0.5q^{-1}},\ H_{22} = H_2 = \frac{1}{1+0.3q^{-1}}$$

Here, V_1 and V_2 are made uncorrelated by making $H_{12} = 0$ and $H_{21} = 0$.

- The operating point for minimum phase transfer function model is as follows:

$$x_{10} = 12.4;\ x_{20} = 12.7;\ x_{30} = 1.8;\ x_{40} = 1.4;\ u_{10} = 3.0;\ u_{20} = 3.0$$

- In the all subsequent discussion, we will considered two channels where channel-1 consisting of input u_1 to output y_1 and channel-2 consisting of input u_2 to output y_2 loop. In Fig.3 channel -2 is closed by unity feedback. The controller block used here is only a gain controller, traditionally known as Proportional Controller. The proportional gain used in above simulation experiment is $K_p = 5$ which is based on the Ziegler Nicholas trial and error method.
- Dither signal shown in Fig.3 is a Generalized Binary Signal (GBN). The average switching time of GBN signal [4] is decided as follows:

$$Avg.Switching\ time = \frac{98\%\text{ of settling time}}{3}$$

- The saturation limits are kept in channel-2. The highest range of the saturation limits are from 2.0 to -3.0 which can be seen by observing the absolute range of the allowable input signals (pump inputs) which is 0 to 5 $Volts$.
- The setpoint y_{2d} is kept at zero for the entire simulation experiment.
- In channel-1, the input u_1 is the PRBS (Pseudo Random Binary Signal) signal. The frequency range of the PRBS is from $[0 \quad b]$ where, b is calculated as follows:

$$b = \frac{\frac{\pi}{\tau}}{\omega_N}$$

Here, Time Constant, $\tau = 5T_s$, Nyquist Frequency, $\omega_N = \frac{\omega_s}{2}$ and Sampling Frequency, $\omega_s = 2\pi f_s = \frac{2\pi}{T_s}$
The amplitude range of the PRBS signal is from -1.0 to 1.0.

- As shown in Fig.3, e_1 and e_2 are zero mean, white noise signals which are passing through the noise filters H_1 and H_2. The noise variances for the signal $e_1$and e_2 are 0.04 for this simulation experiment. The above noise variances are decided on the basis of 2% error in the output variables.
- Kindly note that the simulation experimental time is for $10,000$ seconds.

In the simulation exercise, the dither level are varied in four stages: No Dither, -0.1 to $+0.1$, -0.2 to $+0.2$ and

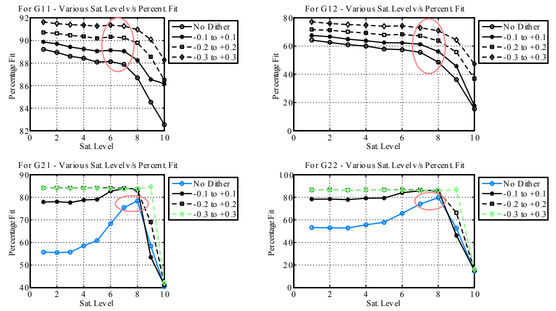

Fig. 4. Percentage fits plotted against various sat. levels

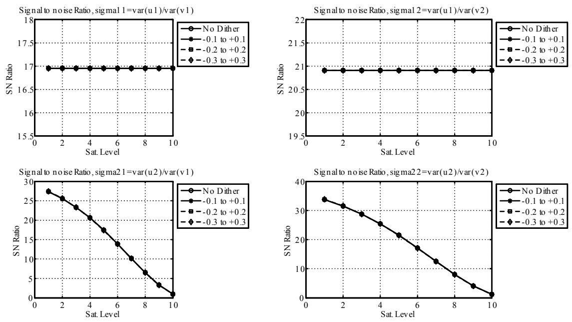

Fig. 5. Signal-to-noise ratio v/s Sat. Levels

-0.3 to $+0.3$. The maximum total variation of dither signal is 12% of the maximum allowable input signal range. The saturation non-linearity is added after the controller block which resembles the constraint controller. Saturation non-linearity is also increased in ten different stages such as 2 to -3, 1.8 to -2.7, 1.6 to -2.4, 1.2 to -1.8, 1.0 to -1.5, 0.8 to -1.2, 0.6 to -0.9, 0.4 to -0.6, 0.2 to -0.3. There are total number of 40 simulation experiments conducted by taking different combinations of dither and saturation levels.

Figure **??** shows the percentage fit of various dynamics of quadruple tank against different saturation levels. Here, when saturation level increases the percentage fit for the dynamics G_{11} and G_{12} decreases. However, for the dynamics G_{21} and G_{22}, as saturation level increases (invoking more nonlinearity), the percentage fit graph becomes non-monotonic. Percentage fit is highest at a point where saturation levels are 0.8 to -1.2 and 0.6 to -0.9 as shown by the highlighted portion (ref. oval) in the Fig.**??** . The important conclusion can be brought here is that invoking nonlinearity helps in breaking correlations between noise and input signal which induces due to feedback but at the same time restricting

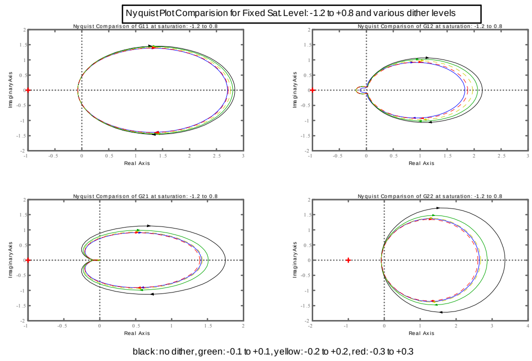

Fig. 6. Nyquist Comparision for fix sat. level (-1.2 to 0.8) and various dither levels

the amount of input signal going to the plant and thus seriously decreasing variability of the input signal beyond certain point. The further improvement in percentage fit can be done by using appropriate dither signal as shown in Fig. **??** for all dynamics.

The signal-to-noise ratio is defined as the ratio of the input signal of interest to the corresponding noise signals. For example, signal-to-noise ratio between the input signal $u_1$and noise v_1 is defined as $\sigma_{11} = \frac{var(u_1)}{var(v_1)}$. In Fig.5, various signal-to-noise ratios $\sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22}$ are plotted against different saturation levels. The observation shows that σ_{11}, σ_{12} remains almost constant but σ_{21}, σ_{22} are gradually decreasing as saturation level increases. However, dither do not play much role here. The second important conclusion can be brought here is that nonlinearity can not be increased to an extent that it severely deteriorate the signal-to-noise ratio.

Figure $6, 7$ shows the Nyquist and Step response comparison respectively when the saturation level is fixed at a point 0.8 to -1.2 and dither levels are varied in four stages described above. Nyquist plot for the highest dither level (red color) is very close to the actual (blue color). The same argument is further supported by the step response plots.

The effect of saturation on bias properties of the plant estimates can be studied from the Fig. 8, 9, 10, 11, 12, 13, 14, 15 which shows the comparison of Nyquist and Step responses with the actual responses for various dynamics when there is no dither signal applied and only saturation levels are changed. The observation shows that for G_{11}, bias remains same for all cases but for G_{12}, there is a gradual increase in bias as saturation increases. For G_{21} and G_{22}, the bias behaves in non-monotonic fashion, it reduces only at a point of focus shown in Fig. 14.

Figure 16, 17 shows the spectrum of input signal u_2 when

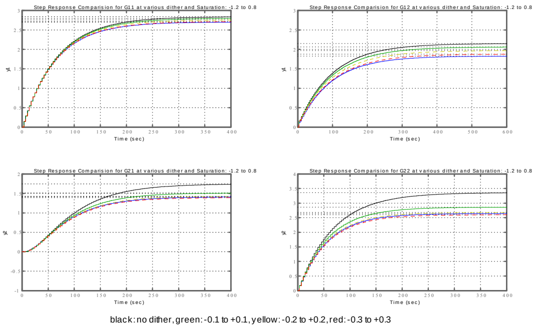

Fig. 7. Step Response Comparision for Fix Sat. Level (0.8 to -1.2) and Various Dither Levels

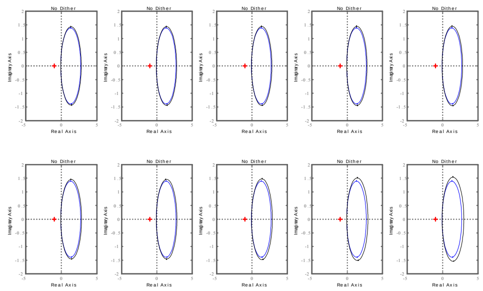

Fig. 8. Nyquist Comparision for G11 (No Dither, Various Sat. Levels)

no dither is applied and Fig. 18, 19 shows the spectrum of input signal u_2 when the highest dither level (i.e. -0.3 to $+0.3$) is applied. Ideally the spectrum of the input signal should similar to a low pass filter response otherwise the high frequency noise can get approximated while doing identification. The results shows that both saturation and dither are useful in shaping the spectrum.

Figure 20, 21, 22, 23 shows the cross correlation between v_2 and u_2 at various dither levels respectively. The observation supports the argument that correlation between noise and input signal can be broken by invoking non-linearity especially when the loop operates under closed-loop configuration.

IV. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

- Invoking non-linearity by using non-linear controller in the closed-loop can contribute in breaking correlation between noise and input signal (manipulated input by controller) induced due to feedback.

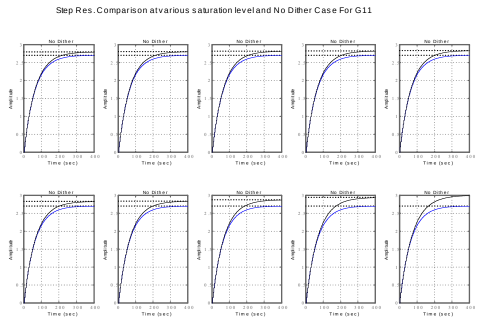

Fig. 9. Step Response Comparision for G11 (No Dither, Various Sat. Levels)

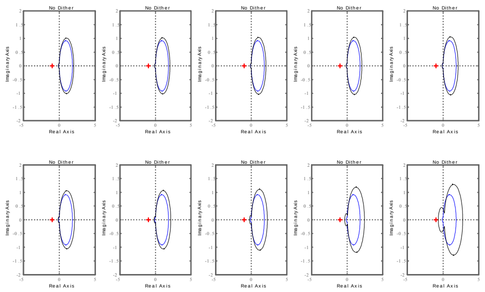

Fig. 10. Nyquist Comparision for G12 (No Dither, Various Sat. Levels)

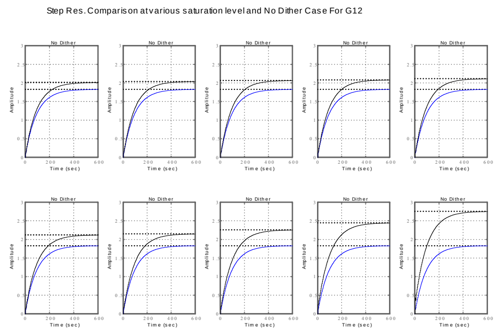

Fig. 11. Step Response Comparision for G12 (No Dither, Various Sat. Levels)

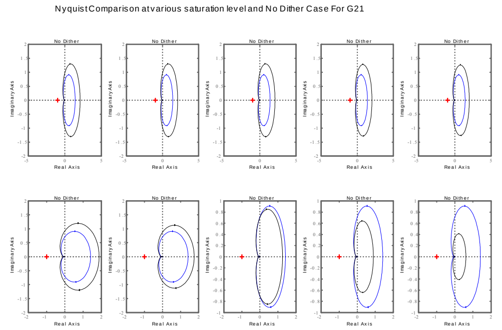

Fig. 12. Nyquist Comparision for G21 (No Dither, Various Sat. Levels)

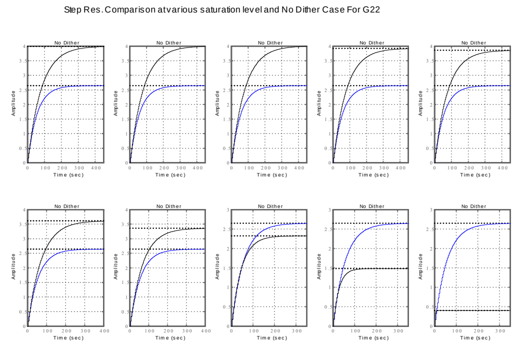

Fig. 15. Step Response Comparision for G22 (No Dither, Various Sat. Levels)

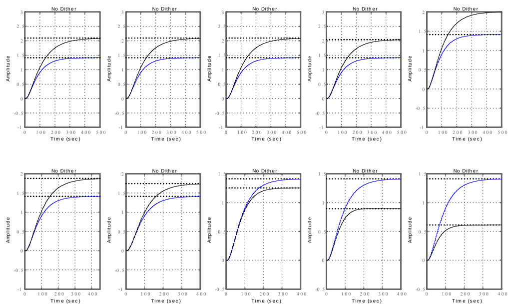

Fig. 13. Step Response Comparision for G21 (No Dither, Various Sat. Levels)

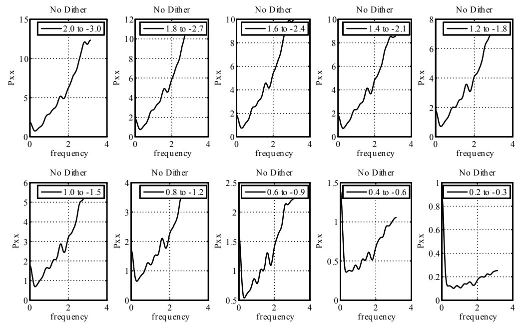

Fig. 16. Spectrum of u2 in Style-1 (No Dither, Various Sat. Levels)

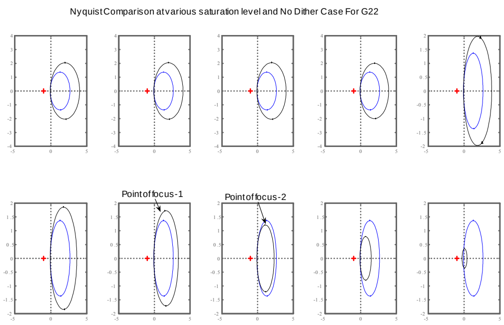

Fig. 14. Nyquist Comparision for G22 (No Dither, Various Sat. Levels)

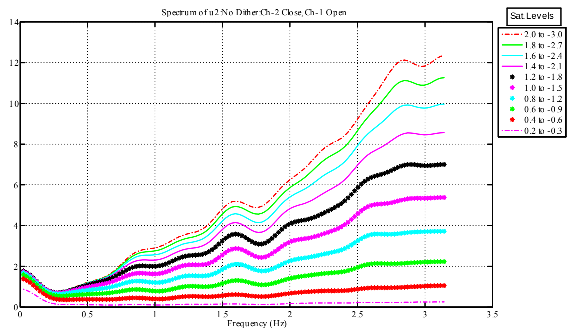

Fig. 17. Spectrum of u2 in Style-2 (No Dither, Various Sat. Levels)

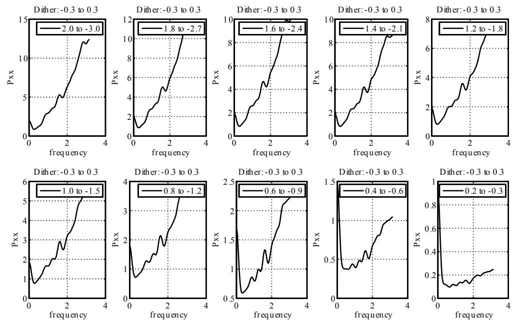

Fig. 18. Spectrum of u2 in Style-1 (Dither: -0.3 to +0.3, Various Sat. Levels)

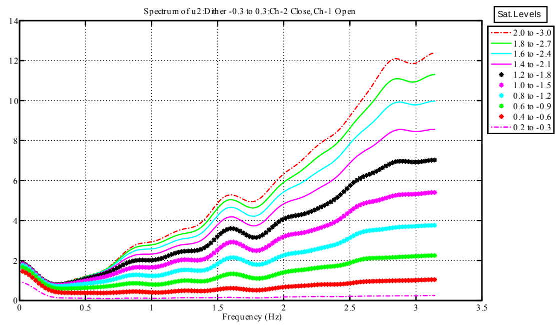

Fig. 19. Spectrum of u2 in Style-2 (Dither: -0.3 to +0.3, Various Sat. Levels)

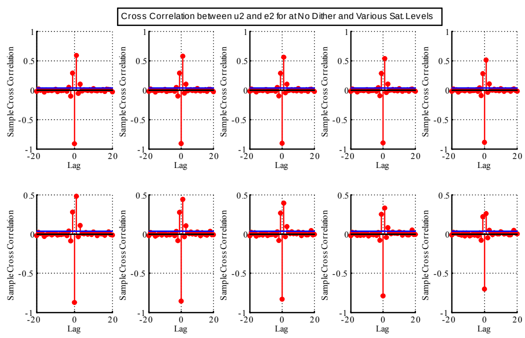

Fig. 20. Cross Correlation between v2 and u2 (No Dither, Various Sat. Levels)

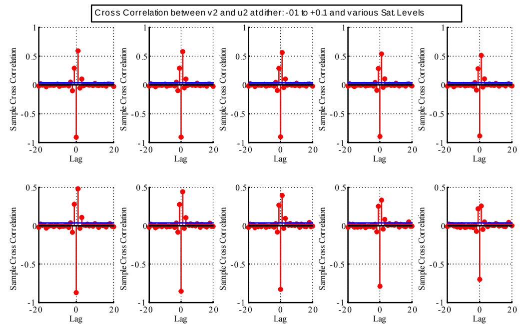

Fig. 21. Cross Correlation between v2 and u2 (Dither: -0.1 to +0.1, Various Sat.Levels)

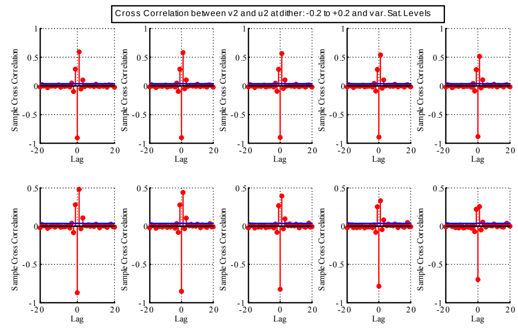

Fig. 22. Cross Correlation between v2 and u2 (Dither: -0.2 to +0.2, Various Sat. Levels)

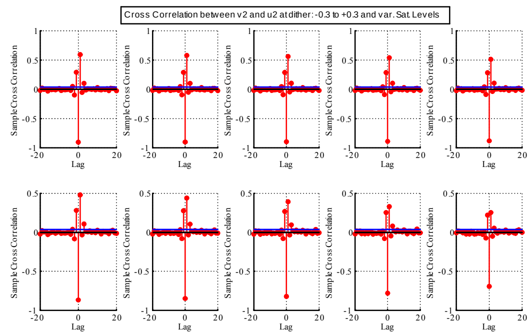

Fig. 23. Cross Correlation between v2 and u2 (Dither: -0.3 to +0.3, Various Sat. Levels)

- Interplay between dither and non-linear saturation can contribute to improvement in model.
- Non-monotonic behavior of model estimation is observed with tightening of constraints due to balancing between reduction in control and lack of excitation resulted from tightening of constraints.

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