New Approach for Two-phase Method to Solve Linear Programming Problem

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Abstract--- In this paper, an alternative method for Two-phase method is introduced. This method is easy to solve linear programming problem. It is most powerful method to reduce number of iteration and save valuable time.

Keywords--- Linear Programming Problem, Optimal Solution, Two-Phase Method, ALPP

I. INTRODUCTION

Dantzig, Orden and Wolfe [1] suggested the process of eliminating artificial variables is performed in Phase I of the solution and Phase II is to get an optimal solution. Khobragade et al. [2, 3, 4] suggested an alternative approach to solve linear programming problem.

In this paper, an attempt has been made to solve linear programming problem (LPP) by new method which is an alternative for Two-phase method. This method is different from Khobragade et al. method.

II. AN ALTERNATIVE ALGORITHM FOR TWO-PHASE METHOD

To find optimal solution of any LPP by an alternative method for Two-phase method, algorithm is given as follows:

Phase I: Simplex method is applied to specially constructed auxiliary linear programming problem (ALPP) leading to final simplex table containing a basic feasible solution to the original problem.

- Step 1. Express LPP in the standard form by introducing slack, surplus and artificial variables.
- Step 2. Assign a cost -1 to each artificial variable and a cost 0 to all other variables (in place of their original cost) in objective function.
- Step 3. Construct the ALPP in which new objective function $Z^*$is to be maximized subject to given set of constraints.
- Step 4. Solve this ALPP by an alternative simplex method. Choose greatest coefficient of decision variables.

  (i) If greatest coefficient is unique, then variable corresponding to this column becomes incoming variable.

  (ii) If greatest coefficient is not unique, then use tie breaking technique.
- Step 5. Compute the ratio with X_B. Choose minimum ratio, then variable corresponding to this row is outgoing variable. The element corresponding to incoming variable and outgoing variable becomes pivotal (leading) element.
- Step 6. Use usual simplex method for this table and go to next step.
- Step 7. Ignore corresponding row and column. Proceed to step 4 for remaining elements and repeat the same procedure until an optimal solution is obtain or there is an indication for unbounded solution.
- Step 8. If all rows and columns are ignored, then current solution is an optimal solution.

Phase II: Now assign the actual costs to the variables in objective function and the zero set to every artificial variable that appears in the basis at zero level. This new

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objective function is now maximized by simplex method to the given constraints. (i.e. simplex method is applied to modified simplex table obtained at the end of Phase I). The artificial variables which are non-basic at the end of Phase I are removed.

III. SOLVED PROBLEMS

Problem 1: Maximize $\boldsymbol{Z = 5x_1 - 2x_2 + 3x_3}$

Subject to $2x_1 + 2x_2 - x_3 \geq 20$

$3x_1 - 4x_2 \leq 3$

$x_2 + 3x_3 \leq 5.$

$x_1, x_2, \quad x_3 \geq 0.$

Solution: LPP is in standard form:

Maximize $Z = 5x_1 - 2x_2 + 3x_3$

Introducing the slack/surplus variables $s_1, s_2, s_3 \geq 0$ and artificial variable $a_1 \geq 0$, the constraints of the above problem becomes:

Subject to $2x_1 + 2x_2 - x_3 - s_1 + a_1 = 2$

$3x_1 - 4x_2 \quad + s_2 = 3$

$x_2 + 3x_3 \quad + s_3 = 5$

$x_1, x_2, x_3, s_1, s_2, s_3, a_1 \geq 0.$

Phase I: Auxiliary L.P. problem is: Maximize $Z = 0x_1 - 0x_2 + 0x_3 + 0s_1 + 0s_2 + 0s_3 - 1a_1$ subject to above given constraints. Solution table for auxiliary problem is as follows:

Table 1: Simplex table

C_B	BVS	X_B	x_1	x_2	x_3	s_1	s_2	s_3	a_1	Ratio
-1	a_1	2	**2**	2	-1	-1	0	0	1	1 →
0	s_2	3	3	-4	0	0	1	0	0	1
0	s_3	5	0	1	3	0	0	1	0	5
0	x_1	1	1	1	-1/2	-1/2	0	0	1/2	
0	s_2	0	0	-7	**3/2**	3/2	1	0	-3/2	0 →
0	s_3	5	0	1	3	0	0	1	0	5/3
0	x_1	1	1	-4/3	0	0	1/3	0	0	
0	x_3	0	0	-14/3	1	1	2/3	0	-1	
0	s_3	5	0	15	0	3	2	1	3	1/3 →
0	x_1	13/9	1	0	0	-4/15	7/45	4/45	4/15	
0	x_3	14/9	0	0	1	1/15	2/45	14/45	-1/15	70/3→
0	x_2	1/3	0	1	0	-1/15	-2/15	1/15	1/5	
0	x_1	23/3	1	0	4	0	1/3	4/3	0	
0	s_1	70/3	0	0	15	1	2/3	14/3	-1	
0	x_2	5	0	1	3	0	0	1	2/15	

Since all rows, columns are crossed and no artificial variable in the basis, hence an optimum solution to the auxiliary problem has been reached.

Phase II: Consider actual costs associated with the original variables, the objective function thus becomes: Maximize $Z = 5x_1 - 2x_2 + 3x_3 + 0s_1 + 0s_2 + 0s_3$

Now apply simplex method in usual manner.

Table 2

C_B	BVS	X_B	x_1	x_2	x_3	s_1	s_2	s_3	a_1	Ratio
5	x_1	23/3	1	0	4	0	1/3	4/3	0	
0	s_1	70/3	0	0	15	1	2/3	14/3	-1	
-2	x_2	5	0	1	3	0	0	1	2/15	

Since all rows and columns are crossed, hence an optimum solution has been reached. Therefore optimum solution is:

$x_1 = \frac{23}{3},\ x_2 = 5,\ x_3 = 0.$ Max. $Z = \frac{85}{3}.$

Problem 2: Maximize $Z = 4x_1 + 5x_2 - 3x_3 + 50$

Subject to $x_1 + x_2 + x_3 = 10$

$x_1 - x_2 \quad \geq 1$

$2x_1 + 3x_2 \quad \leq 40.$

$x_1, x_2, x_3 \geq 0.$

Solution: LPP is in standard form:

Maximize $Z = 4x_1 + 5x_2 - 3x_3 + 50$

Introducing the surplus variable $s_1 \geq 0$ and artificial variables $a_1, a_2 \geq 0$, the constraints of the above problem becomes:

$$\text{Subject to } x_1 + x_2 + x_3 + a_1 = 10$$
$$x_1 - x_2 - s_1 + a_2 = 1$$
$$2x_1 + 3x_2 + s_2 = 40.$$
$$x_1, x_2, \quad x_3, s_1, a_1, a_2 \geq 0.$$

Phase I**:** Auxiliary L.P. problem is: Maximize $Z =$ *0x1+0x2−0x3+50+0s1−1a1−1a2* subject to above given constraints. Solution table for auxiliary problem is as follows:

Table 3: Simplex Table

C_B	BVS	X_B	x_1	x_2	s_1	x_3	a_1	a_2	Ratio
0	x_3	10	1	**1**	0	1	0	0	10→
-1	a_1	1	1	-1	-1	0	1	0	-
-1	a_2	40	2	3	0	0	0	1	40/3
0	x_2	10	1	1	0	1	0	0	
-1	a_1	11	**2**	0	-1	1	1	0	11/2 →
-1	a_2	10	-1	0	0	-3	0	1	
0	x_2	9/2	0	1	1/2	-1/2	-1/2	0	
0	x_1	11/2	1	0	-1/2	1/2	1/2	0	
-1	a_2	31/2	0	0	-1/2	-5/2	1/2	1	

Since all rows, columns are crossed and remaining element is negative, hence an optimum solution to the auxiliary problem has been reached. But at same time artificial variable a_2 appears in the basic solution at a position level. Hence the given LPP has no feasible solution. Here there is no need to enter Phase II.

Problem 3: Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$

$$\text{Subject to } x_1 + 2x_2 + 3x_3 = 15$$
$$2x_1 + x_2 + 5x_3 = 20$$
$$x_1 + 2x_2 + 2x_3 + x_4 = 10.$$
$$x_1, x_2, \quad x_3, \quad x_4 \geq 0.$$

Solution: LPP is in standard form:

Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$

Introducing the artificial variables $a_1, a_2, a_3 \geq 0$, the constraints of the above problem becomes:

$$\text{Subject to } \mathrm{x_1 + 2x_2 + 3x_3 + a_1 = 15}$$
$$\mathrm{2x_1 + x_2 + 5x_3 + a_2 = 20}$$
$$\mathrm{x_1 + 2x_2 + x_3 + x_4 = 10.}$$
$$\mathrm{x_1, x_2, x_3, x_4, a_1, a_2, a_3 \geq 0.}$$

Phase I: Auxiliary L.P. problem is: Maximize $Z =$ *0x1+0x2+0x3−0x4−1a1−1a2−1a3* subject to above given constraints. Solution table for auxiliary problem is as follows:

Table 4: Simplex table

C_B	BVS	X_B	x_1	x_2	x_3	x_4	a_1	a_2	Ratio
-1	a_1	15	1	2	3	0	1	0	5
-1	a_2	20	2	1	**5**	0	0	1	4 →
0	x_4	10	1	2	1	1	0	0	10
-1	a_1	3	-1/5	**7/5**	0	0	1	-3/5	15/7 →
0	x_3	4	2/5	1/5	1	0	0	1/5	
0	x_4	6	3/5	9/5	0	1	0	-1/5	10
0	x_2	15/7	-1/7	1	0	0	5/7	-3/7	
0	x_3	25/7	3/7	0	1	0	-1/7	2/7	
0	x_4	15/7	**6/7**	0	0	1	-9/7	4/7	5/2 →
0	x_2	5/2	0	1	0	1/6	1/2	-1/3	
0	x_3	5/2	0	0	1	1/2	1/2	0	
0	x_1	5/2	1	0	0	7/6	-3/2	2/3	

Since all rows, columns are crossed and no artificial variable in the basis, hence an optimum solution to the auxiliary problem has been reached.

Phase II: Consider actual costs associated with the original variables, the objective function thus becomes: Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$

Now apply simplex method in usual manner.

Table 5

C_B	BVS	X_B	x_1	x_2	x_3	x_4	a_1	a_2	Ratio
2	x_2	5/2	0	1	0	1/6	1/2	-1/3	
3	x_3	5/2	0	0	1	1/2	1/2	0	
1	x_1	5/2	1	0	0	7/6	-3/2	2/3	

Since all rows and columns are crossed, hence an optimum solution has been reached. Therefore optimum solution is:

$$x_1 = x_2 = x_3 = \frac{5}{2}. \text{ Max. } Z = 15.$$

IV. CONCLUSION

An alternative method for Two-phase method to obtain the solution of linear programming problem has been derived. The proposed algorithm has simplicity and ease of understanding. This reduces number of iterations and save valuable time

REFERENCES

[1] G. B. Dantzig , Maximization of linear function of variables subject to linear inequalities (Koopman Cowls Commssion Monograph, 1951).

[2] K. Lokhande, N. Khobrade and P. Khot, Simplex Method: An Alternative Approach, International Journal of Engineering and Innovative Technology, 3(1), 2013, 426-428.

[3] N. Khobragade , Alternativ Approach to the Simplex Method-I, Bulletin of Pure and Applied Science,23(1),2004, 35-40.

[4] N. Khobragade and P. Khot, Alternativ Approach to the Simplex Method-II, Acta Ciencia Indica,3 , 2005, 651.

[5] S. D. Sharma, Operation research (Kedar Nath Ram Nath).

[6] S. I. Gass, Linear programming (McGraw-Hill Kogakusha, 1969).

[7] K.P. Ghadle, T. S. Pawar and N. Khobragade, Solution of Linear Programming Problem by New Approach, International Journal of Engineering and Innovative Technology,3(6), 2013, 301-307.